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PRINCETON, NEW JERSEY 08540  
609 924-8778

A STUDY OF  
SOME ECONOMIC FACTORS RELATING  
TO THE DEVELOPMENT AND IMPLEMENTATION  
OF A SATELLITE POWER SYSTEM

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ECON, Inc.  
900 State Road  
Princeton, NJ 08540

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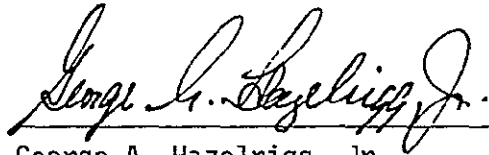
## ABSTRACT

This study examines three areas relating to the design, development and implementation of a satellite power system (SPS): an analysis of the effect of energy R&D programs in general and SPS in particular on optimal fossil fuel consumption patterns, a study of alternative uses of SPS technologies, and a study of the electric power market penetration potential for SPS. In the first area, it is shown that a credible program of R&D on long-range energy alternatives leads to lower optimal prices for fossil fuels, resulting in large short-term benefits accruing to the specific program elements. These benefits should be considered in establishing energy R&D policy. In the second area, a number of different SPS technologies were studied to determine and assess their alternative uses (uses outside the SPS program). Several alternative uses of SPS technologies were identified; however, the markets for these technologies are generally quite diffuse and difficult to assess. The notable exception is solar array technology which has, potentially, a very large non-SPS market. The final area deals with the rate at which SPS units would be demanded as a function of their capital investment cost. It is shown that the market for SPS units derives from two components of demand: the demand created by growth in the electrical energy demand which leads to an increased demand for baseload generating capacity, and a demand created by the need to replace retiring capacity. The latter demand component is likely to be larger than the former unless the nation undertakes a major shift to an all-electric economy over the next 50 years or so. Without such a shift, energy demand growth would probably accommodate less than two SPS units per year until sometime well after the year 2000.

## NOTE OF TRANSMITTAL

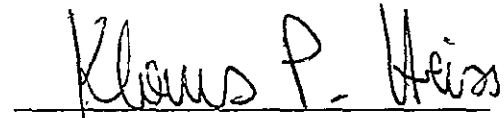
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Submitted by:



George A. Hazelrigg, Jr.  
Director, Systems Engineering

Approved by:



Klaus P. Heiss  
President

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## 1. INTRODUCTION

This report documents the results obtained in a short study of some economic factors relating to the development and implementation of a satellite power system (SPS). The study encompasses three tasks:

1. A study of the effect of an SPS development program on fossil fuel prices
2. A study of the benefits attributable to alternative uses of SPS technologies
3. A study of the electric power market penetration of SPS.

The objective of the study was to identify and assess some economic factors relating to the justification for developing SPS and the appropriate level of funding for the development program, and the implementation of SPS into the electric power market and, thence, appropriate construction rates and implementation benefits.

The first task examines optimal pricing strategies for nonrenewable resources, such as petroleum, with specific application to the pricing of OPEC (Organization of Petroleum Exporting Countries) crude oil. It is shown that the optimum price is affected by research and development programs aimed at the development of substitutes which would ultimately result in a lower demand for petroleum. This effect appears to be a dominant factor in the present OPEC pricing considerations and leads to a very substantial annual benefit for petroleum importing countries. Clearly, only a small fraction of the total benefit thus obtained would be captured by an SPS development program but, nonetheless, this should be one of the major economic considerations in the decision to pursue an SPS.

The second task deals with alternative uses (not "spinoffs") of SPS technologies. Alternative uses of SPS technologies are uses of technologies that must be developed for SPS outside of the SPS program. These are different from spinoffs since a forward-looking, rather than rearward-looking, point of view is taken. A number of technologies are identified and a few, particularly solar cell technology, are analyzed to determine potential markets and, thence, benefits to these markets that could result from SPS technology development.

The third task analyzes the electric utility industry to determine a reasonable SPS construction rate as a function of cost. The demand for SPS would derive from two sources: new baseload energy demand as the result of a growing demand for electricity, and a demand created from the retirement of existing generating capacity. It is likely that the latter of these components of demand will predominate unless there is a major shift to an all-electric economy over the next 50 years or so. The primary competition for SPS in the market over the next 50 years is likely to be nuclear energy from fission.

## 2. OPTIMAL FOSSIL FUEL CONSUMPTION PATTERNS

The major economic rationale made for research and development on long-range energy alternatives to date has focused largely around the potential cost savings that new technologies might offer when they are implemented. This benefit can be referred to as the implementation benefit of a new energy system. The implementation benefit of long-range energy technologies is generally small (or, at best, provides a weak rationale for an R&D program) for two reasons: First, it is unlikely that any long-range energy technology will be ready for commercial implementation for 20 to 30 years from the present and, following that, it will likely be another 20 to 50 years before the new technology has a major impact on the energy marketplace. Thus, any cost savings benefits that might accrue must be discounted over some 30 to 80 years in order to determine their present value. Discounting over this period of time, even for relatively low discount rates, results in a substantially decreased present value. Second, at least during the initial period after first commercial implementation of a new energy system, it is unlikely that the new system will enjoy a significant cost advantage over existing systems and, in fact, there may actually be slight economic disadvantages. This is primarily due to the fact that any new technology has associated with it a learning period during which continuing system developments, in light of actual operating experience, bring the system up to its true economic potential.

There exists, however, a second and very substantive rationale for pursuing long-range energy technologies. This rationale derives from the notion that, given the existing reserves of easily recoverable fossil

fuels to be finite, we would be more willing to consume these reserves in the short term if there is an assurance of energy alternatives in the long term than if long-term alternatives are not likely to exist. This notion applies not only to domestic reserves, but to foreign reserves as well, and it applies to both consumers and producers. In this study, we examine the potential impact of this notion upon the Organization of Petroleum Exporting Countries (OPEC) and the resulting benefits that might be obtained by the United States as the result of accelerated petroleum consumption patterns resulting from a well-considered program of energy R&D that promises long-range energy alternatives, whatever they may be.

With respect to OPEC, the fundamental assumption is made that the operating objective is to maximize the value of the petroleum resource to the holders of that resource. The control variable by which this maximization can be accomplished is the petroleum export price. As shown in Figure 2.1, the net revenues generated by the export of petroleum next year depend on the price at which the petroleum is sold. The quantity of petroleum demanded over this period is given by curve D, the demand function. The marginal cost of providing petroleum is given by curve S, the supply function, and the net revenues generated as a function of price are shown as the shaded areas. If the price were set high [Figure 2.1(a)], the net revenue generated from the sale of each barrel of petroleum would be high, but the total sales would be low and, thus, the total net revenue generated over the year would be low. Clearly, with a very high price, the reserves would last a long time. On the other hand, if the price were set very low [Figure 2.1(b)], the quantity demanded would be quite high but the net revenue generated per barrel would be very low and, again, the

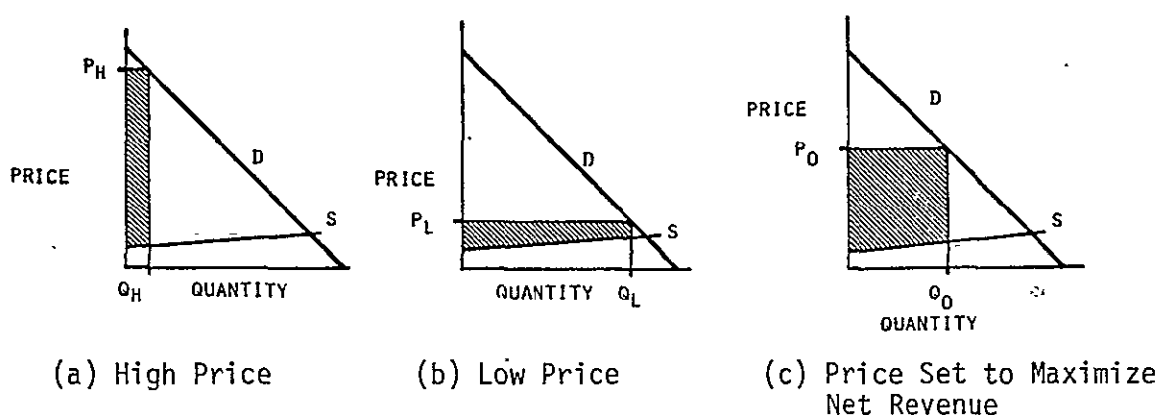


Figure 2.1 Net Revenues to Exporting Nations as a Function of Price for the Next Time Period

total net revenue generated over the year would be low. Also, it now occurs that the reserves are depleted quite rapidly. It is obvious that some price in between these extremes maximizes the total net annual revenue [Figure 2.1(c)].

Despite the above arguments, one might still believe that the best approach would be to set the price high and take a very long time to net a very large revenue. The problem with this solution is that money has a value which is associated with the time at which it is received. That is, a dollar received today is worth more than a dollar received a year from today. This is because, presumably, the dollar could be put to some use today that would yield more than a dollar in one year, inflation aside. Taking the time value of money into account, it is clear that the optimal control is to set the price somewhere between that price which maximizes the total net revenue during the next period and the maximum price at which any petroleum would be demanded.

The problem as now stated is formulated and solved below to yield the optimal OPEC petroleum price as a function of time. If now an energy R&D program is established which has a high expectation of developing an

alternative energy source that could be available in, say, 30 years and, in so being would reduce the demand for OPEC petroleum after that time, the current expectation that this could occur will alter the optimal petroleum price downward and result in increased consumption in the short term. As is discussed later, it is likely that ongoing energy R&D programs are currently depressing the price of OPEC oil and the actual observed effect of increased energy R&D would be a slower rate of increase in OPEC prices in the future.

## 2.1 An Oil Pricing Strategy Model

A model of optimal petroleum<sup>\*</sup> pricing strategies is developed below. The fundamental assumption employed is that the holders of oil reserves wish to price their oil so as to maximize its value to themselves. This assumption leads to a mathematical model of the form:

$$\left. \begin{array}{ll} \text{Max} & \int_0^{\infty} e^{-\rho t} U(Q, q, t) dt \\ \text{subject to:} & \dot{Q} = -q \\ & \text{and } q, Q \geq 0 \\ & \text{where } Q(t=0) = Q_0 \text{ is given} \end{array} \right\} \quad (2-1)$$

The problem, (2-1), is read: select the functional  $q(t)$  which maximizes the present value of present and future utilities,<sup>\*\*</sup>  $U(Q, q, t)$ , (to the resource holders) from oil production subject to the constraints that total petroleum inventory,  $Q$ , diminishes at the rate at which petroleum

<sup>\*</sup>This model applies specifically to nonreplenishable reserves.

<sup>\*\*</sup>By "utility" we mean worth.

is produced for consumption,  $q(t)$ , and neither  $q(t)$  nor  $Q$  can be negative. The initial level of reserves,  $Q_0$ , is given.  $\rho$  is the discount rate. If we take utility to be expressed by the flow of net revenues (revenues minus costs), then the problem is one of maximizing the present value of net revenue accruing from the initial reserves,  $Q_0$ . As such, it is a dynamic profit-maximization problem. This problem is solvable using the calculus of variations.

Suppose now that there exists, through energy R&D, a future possibility of a "substitute" for petroleum.\* The effect of a substitute would be to lower the demand for petroleum, according to its supply characteristics, after the date at which it is first available. The change in demand affects the net revenue function and thus also the optimal consumption pattern. Let us say that the date at which a substitute technology becomes available, denoted  $\tau$ , is uncertain, having a probability distribution  $\omega_\tau$ ; that is,  $\tau$  is a random variable. The form and parameters of this distribution function on  $\tau$  are knowable and we will acknowledge that they are influenced by R&D investments in the technology. Then, changes in optimal resource consumption resulting from changes in  $\omega$  are properly attributable to the R&D expenditures.

Let us define the variable  $W(Q_\tau)$  as the maximum present value (discounted to time  $\tau$ ) of the reserves,  $Q_\tau$ , which are in existence at time  $\tau$ , the date the new substitute becomes available, subject to the same constraints as in (2-1), that is,

---

\* By substitute we mean any alternative energy source that will result in a reduced demand for petroleum.

$$\left. \begin{aligned}
 W(Q_\tau) &= \text{Max} \int_{\tau}^{\infty} e^{-\rho(t-\tau)} U(Q, q, t) dt \\
 &\text{Subject to} \\
 &\quad \dot{Q} = -q \\
 &\quad q, Q \geq 0
 \end{aligned} \right\} \quad (2-2)$$

Given any  $Q_\tau$ , the maximization process provides a unique functional  $q(t)$  and, thus,  $W(\cdot)$  is not a function of  $q(t)$ .

Given the above definitions (and the assumption that the resource holders maximize the expected present value of their reserves), the possibility of a substitute technology means that (2-1) can now be expressed as:

$$\begin{aligned}
 \text{Max } E \left\{ \int_0^{\infty} e^{-\rho t} U(Q, q, t) dt \right\} \\
 = \text{Max} \int_0^{\infty} \omega_\tau \left\{ \int_0^{\tau} e^{-\rho t} U(Q, q, t) dt + e^{-\rho \tau} W(Q_\tau) \right\} d\tau
 \end{aligned} \quad (2-3)$$

Define

$$\Phi_t = \int_t^{\infty} \omega_\tau d\tau = 1 - \int_0^t \omega_\tau d\tau = 1 - \Omega_t \quad (2-4)$$

where  $\Omega_t$  is the cumulative distribution function associated with  $\omega$ . Integrating (2-3) by parts,

$$\begin{aligned}
 E \left\{ \int_0^{\infty} e^{-\rho t} U(Q, q, t) dt \right\} \\
 = \int_0^{\infty} e^{-\rho t} \left\{ U(Q, q, t) \Phi(t) + \omega(t) W(Q) \right\} dt
 \end{aligned} \quad (2-5)$$

Thus, we have the problem:



$$\left. \begin{array}{l} \text{Maximize (2-5) subject to} \\ \dot{Q} = -q \\ q, Q \geq 0 \end{array} \right\} \quad (2-6)$$

Note that the optimal policy is to pursue the solution path to (2-6) until the new technology becomes available (at time  $\tau$ ). From  $\tau$  forward, the optimal solution path is that which yields  $W(Q_\tau)$ .

In general, (2-6) is solvable, but with considerable difficulty. However, Dasgupta and Heal<sup>\*</sup> have shown for a similar problem that, if one introduces certain restricting conditions, the problem resolves to one which is less difficult to solve. In particular, consider the problem

$$\text{Max } E \left\{ \int_0^\infty e^{-\delta t} U(Q, q, t) dt \right\} \quad (2-7)$$

subject to the constraints in (2-6). This problem has the identical solution to (2-6) given the following restrictions.

$$\begin{aligned} \text{(i)} \quad & \frac{\partial W}{\partial Q} = 0 \\ \text{(ii)} \quad & \delta_t = \rho t + \int_0^t \psi_\theta d\theta = \rho t + \psi_t \end{aligned}$$

(See the referenced article for proof.) The problem involving uncertainty has thus been translated into a deterministic problem with a suitably chosen discount factor,  $\delta_t$ . The amount,  $\psi_t$ , which has to be added to the discount rate  $\rho$  is typically time variant and introduces difficulties

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<sup>\*</sup>Dasgupta, P. and G. Heal, "The Optimal Depletion of Exhaustible Resources," The Review of Economic Studies, 1974 Symposium, pp. 3-28.

in obtaining the solution path. If, however, one assumes that  $\omega_t$  takes on an exponential distribution, that is,

$$\omega_t = \pi e^{-\pi t}$$

then  $\psi_t$  is constant and equal to  $\pi$ , where  $\pi$  is the inverse of the mean value of  $\omega_t$ . One can then simply solve (2-1), adding increments to  $\rho$  as the expected time to availability of the substitute technology decreases.

## 2.2 Assumptions Employed

The solution to (2-1) is derived in Appendix A. Before discussing parameter estimation and investigating the results, it is useful to say something more about the assumptions employed. The first and most crucial assumption to this entire analysis is that resource holders employ a decision rule which maximizes the expected net present value of their resources to themselves. This assumption may be critiqued from at least the following angles:

1. Do the resource holders employ any rational decision rule at all?
2. Assuming a rational decision rule is used, is it or should it be based on maximization of expected net present value?
3. How does one establish an appropriate discount rate for decision making purposes?
4. Will the resource holders react to changes in energy R&D programs?

While these questions cannot be answered definitively here, some comments are in order. First, observe that, while there is no guarantee that the resource holders now apply, or will in the future apply, a rational decision rule of any sort, it is clearly to their advantage to do so. Because this is so, one might expect temporary deviations from an "optimal" course of behavior; however, it is recognized that economic pressures exist

tending to force behavior toward the optimal solution. By the same reasoning, we would expect a reaction to significant changes in energy R&D policy. Whether such changes do or do not occur is left open for debate. Strong support to the hypothesis that OPEC policy-makers are sensitive to U.S. energy policy has recently been furnished by the Shah of Iran who, in a recent interview, said that his country would give the United States a price break, "...provided you also take the necessary measures and steps for conservation and finding new sources of energy."<sup>\*</sup> The key point here is that the optimum decision rule defines a good estimate of the true magnitude of the benefit to be achieved from energy R&D. Beyond this, it should be observed that proper temporal distribution of a resource also benefits the consumers in the long run. Thus, it is possible that the consumers may suffer a loss if the resource holders operate sufficiently off of an optimal logic. Representing the objective function of the decision rule as the expected net present value of the resource neglects the possibility of the decision makers to take positions of risk averseness or proverseness. Usually, decision makers are, if not expected value decision makers, risk averse. This position would increase the benefit of energy R&D. However, because the decision makers are themselves increasingly dependent on energy, from the point of view of the decision rule, they might actually choose to take a risk proverse position which could lower the benefit of energy R&D. Finally, there is the issue of the discount rate. Again, without answering the question, we would simply

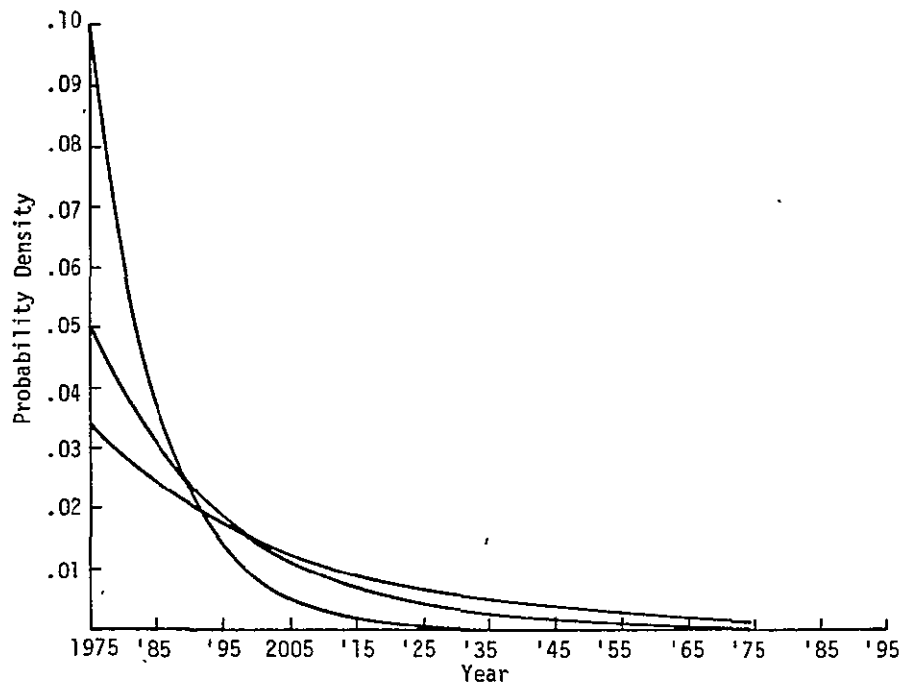
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<sup>\*</sup> Interview with the Shah of Iran, telecast on ABC's "Issues and Answers," November 20, 1977. See Appendix B for a transcript of this interview.

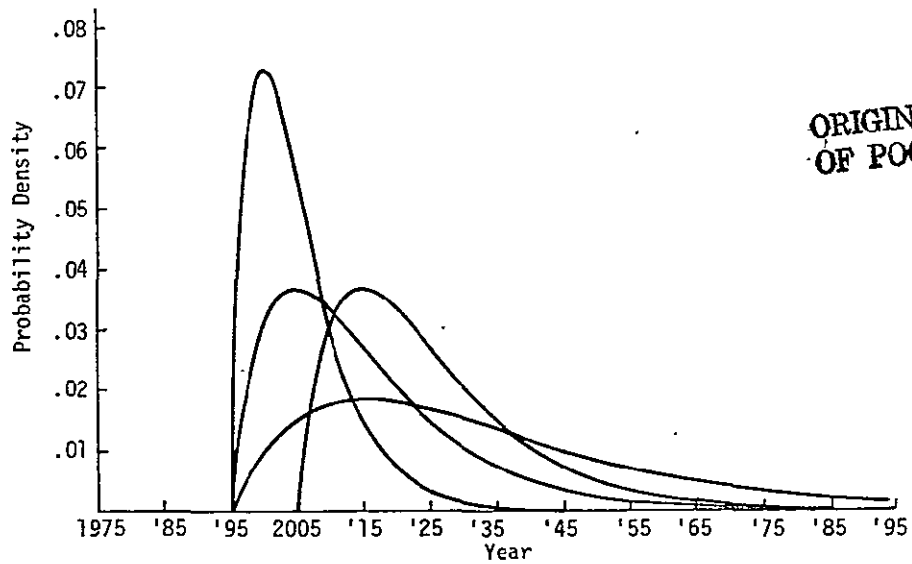
comment that it is sensible for the discount rate of a less developed nation to be lower than that of a developed nation insofar as the less developed nation would probably not have available the equivalent range of investment opportunities. And it is probably appropriate that the correct discount rate be bounded by the nation's rate of technology innovation. In the United States, over the past 100 years or so, this rate has been about three percent.

The second assumption,  $\frac{\partial W}{\partial Q} = 0$ , can be derived from the assumption that the reserves remaining at the time of availability of the new technology are valueless and contribute nothing more to the total utility. Such an assumption tends to accelerate consumption of the reserves and thus leads to an overstatement of the benefit, given that the assumption is too strong. It is, of course, unlikely that any technology would produce free energy and thus petroleum would most likely retain at least some value for the foreseeable future. This assumption also has the effect of reducing the importance of near-term arrival date reductions (for example, from 33 to 20 years from present) relative to reductions in the longer term time frame (for example, from 100 to 50 years from present).

The third assumption requires, for simplicity in the computations, the date of arrival of the substitute to take on a particular probability distribution. The probability distribution assumed for the analysis conducted here is of the family shown in Figure 2.2(a). These distributions result from the assumption that the probability of arrival of the new technology is the same over any unit interval of time given that the new technology has not yet arrived (that is, it is a Poisson distributed random variable). This situation probably does not reflect the true state



(a) Family of Exponential Distributions



(b) Family of Modified Gamma Distributions

Figure 2.2 Probability Distributions for Arrival  
Date of Substitute Technology

of high technology energy alternatives, and certainly not SPS. The "arrival" date in which we are interested is the time of general commercial availability on a wide scale. Surely this is at least twenty years away when the time for SPS construction is considered. The probability also appears to increase up to some point, decreasing thereafter. The general form of the probability density function  $\omega_t$  in which we are interested is probably more like the type shown in Figure 2.2(b). Here the probability of the availability of the alternative is zero up to a certain point and takes on a gamma distribution thereafter. The point in time at which an alternative becomes possible, as well as the mean of the gamma distribution, are parameters affected by energy technology R&D.

A fourth assumption employed is on the form of the utility function. In particular, an exponential demand growth for OPEC petroleum has been assumed. This growth pattern is not likely to sustain indefinitely into the future but, probably, the assumption is adequate for this analysis.

The assumptions discussed above introduce considerable simplification into the present analysis. Thus, the results obtained should be viewed strictly as an order-of-magnitude indication of the potential size of the benefits attributable to a program of long-range energy R&D. It is yet necessary to separate out the benefits of SPS research as one component of an overall program. This is done in Section 2.5 below.

### 2.3 Functional Forms and Parameters Estimation

The R&D benefits discussed here apply to the case of any finite energy resource. However, since the United States is mainly an importer of

petroleum, which is imported from the OPEC\* nations, and since we wish to quantify the benefits to the United States due to the effect of energy R&D on energy prices, we focus now on optimal pricing of petroleum from OPEC's point of view. The importer petroleum demand and OPEC petroleum extraction cost functions are discussed in this section.

For convenience, a linear demand function for crude oil shifting out exponentially over time was assumed. It takes the form:

$$p(t) = a_1 + a_2 e^{\gamma t} q(t) \quad (2-8)$$

It is proposed that the 1974 price/quantity position (approximately 11.3 billion barrels annually at \$10 per barrel) represents an equilibrium position and thus the 1974 demand curve is presumed to pass through that point. Given a long-run elasticity of demand for imports and the 1974 price/quantity point, the demand function can be fully specified using Equations (2-8) and (2-9):

$$a_2 = \frac{1}{E} \left( \frac{p_{1974}}{q_{1974}} \right) \quad (2-9)$$

where E is the estimated long-run demand elasticity. For this analysis, E is assumed to have a value of -1.0 in the "base" case; this is felt to be a reasonably good estimate of its true value. Current estimates of the long-run elasticity of world demand for petroleum run in the range of -0.4 to -0.8.\*\* Demand for OPEC oil in particular can be expected to be

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\* Organization of Petroleum Exporting Countries.

\*\* See, for example, M. Kennedy, "A World Oil Model," in Econometric Studies of U.S. Energy Policy, D. W. Jorgenson (ed.), American Elsevier, New York, 1976; and P. Verleger, J. Osten and D. Miller, "Trends in the World Price of Oil: 1975-1980," Data Resources, Inc., 1975.

somewhat more elastic. Thus, elasticities in the range of -0.5 to -1.0 are reasonable.

The cost of extraction,  $C_t$ , is taken from Kalymon.\* It is presumed to be quadratic in the rate of extraction,  $q$ , and linear in the quantity of remaining reserves.

$$c(t) = b_1 q(t) + \frac{b_2}{2} q^2(t) + b_2 [Q_0 - Q(t)] q(t) \quad (2-10)$$

$Q_0$  is the initial level of OPEC reserves and Kalymon's estimate of 800 billion barrels is used.  $b_1$  and  $b_2$  are estimated by Kalymon as 0.12 and 0.005, respectively. The reader is referred to the referenced article for more discussion of these estimates.

Net revenue is used as a measure of utility in (2-1). For any particular point in time, utility is defined, using (2-8) and (2-10), as:

$$\begin{aligned} U(t) &= p(t) q(t) - c(t) \\ &= (a_1 - b_1 - b_2 Q_0) q(t) + (a_2 e^{\gamma t} - \frac{b_2}{2}) q^2(t) + b_2 Q(t) q(t) \end{aligned} \quad (2-11)$$

Again, the decision rule used is to maximize the expected present value of utility, as expressed in (2-11).

## 2.4 Analytical Results

The results of the analysis are strikingly close to the actual OPEC pricing policies. With an expected alternative arrival date of 50 years, the optimal 1974 price and quantity estimated by this simple model is

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\*B. Kalymon, "Economic Incentives in OPEC Oil Pricing Policy," Journal of Development Economics, 2(4), December 1973, pp. 337-362.



\$11.10 per barrel and 10 billion barrels annual exports for the base case;<sup>\*</sup> nearer expected arrival dates yield higher exports at lower prices. This compares with the actual OPEC decision of about \$10.0 per barrel and 11.3 billion barrels annual export in 1974. The actual OPEC decisions are, of course, tempered by political influences (for example, arms sales) which are not incorporated into this decision model and which would tend to depress the price from its strictly economic optimum.

Figure 2.3 shows the price path and inventory drawdown for the base case with no alternative available. Figure 2.4 shows the corresponding price-quantity trajectory over time for the no alternative case. Note that given the assumptions, prices continue to rise (roughly exponentially) and exports peak at 22.2 billion bbl in about 2008, then decline rapidly after that. Figure 2.5 shows the same plot with an expected alternative arrival date of 50 years. In comparison, this case shows the export rate has accelerated, peaking at 24.5 billion bbl in 2005. Figure 2.6 shows in one graph the price and depletion trajectories for several expected arrival dates for the base case.

For each increment of time, the expected benefits are calculated as the consumers' surplus attributable to the price drop (from the no-alternatives case,  $P_1$ , to the availability-of-substitutes case,  $P_2$ , in Figure 2.7) multiplied by the probability that the alternative has not already been developed. Integrating expected discounted benefits across time then

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<sup>\*</sup>The "base case" assumes an OPEC demand growth rate of 5 percent per year, OPEC discount rate of 4 percent and an export demand elasticity of -1.0.

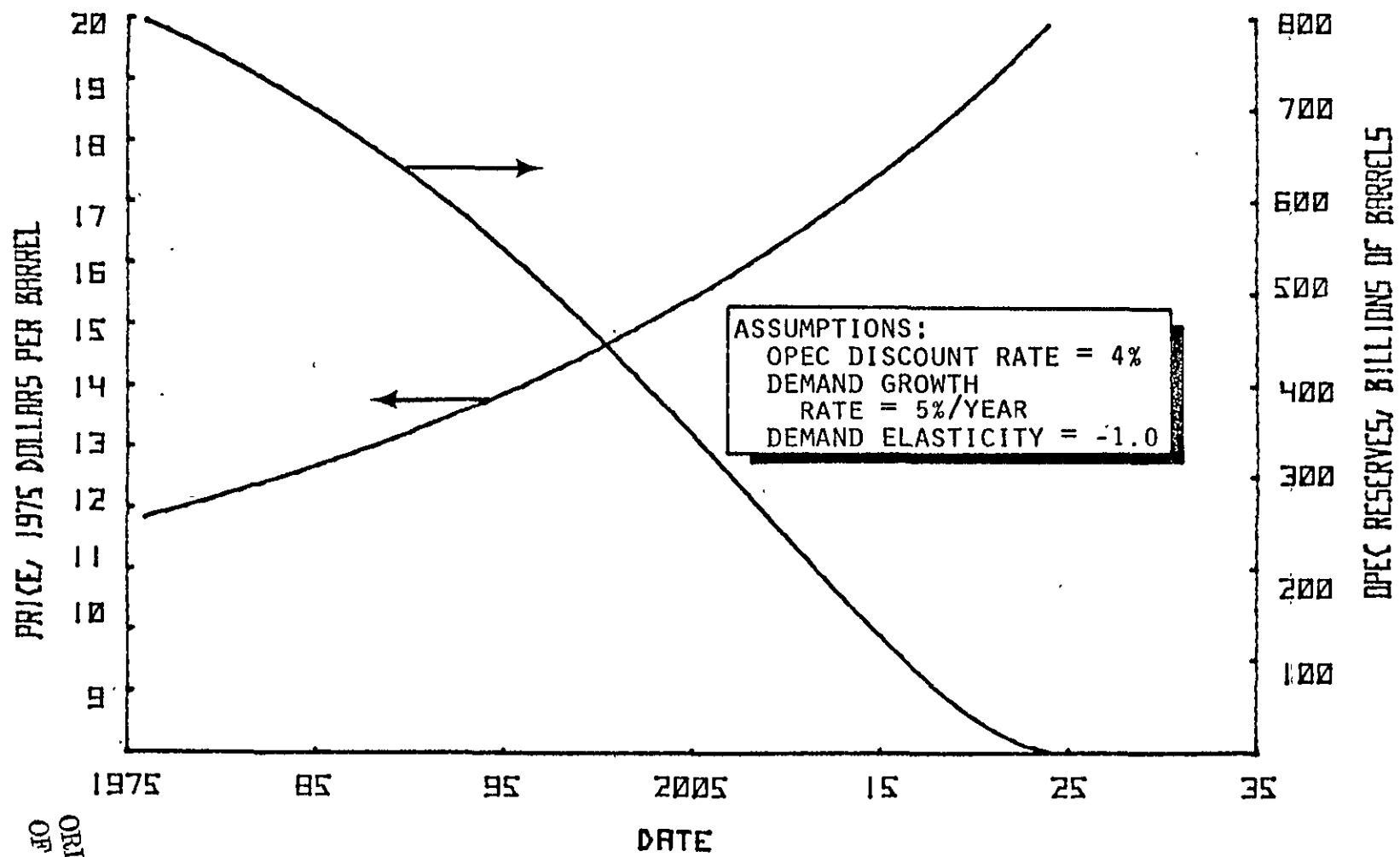


Figure 2.3 An Optimal Consumption Pattern

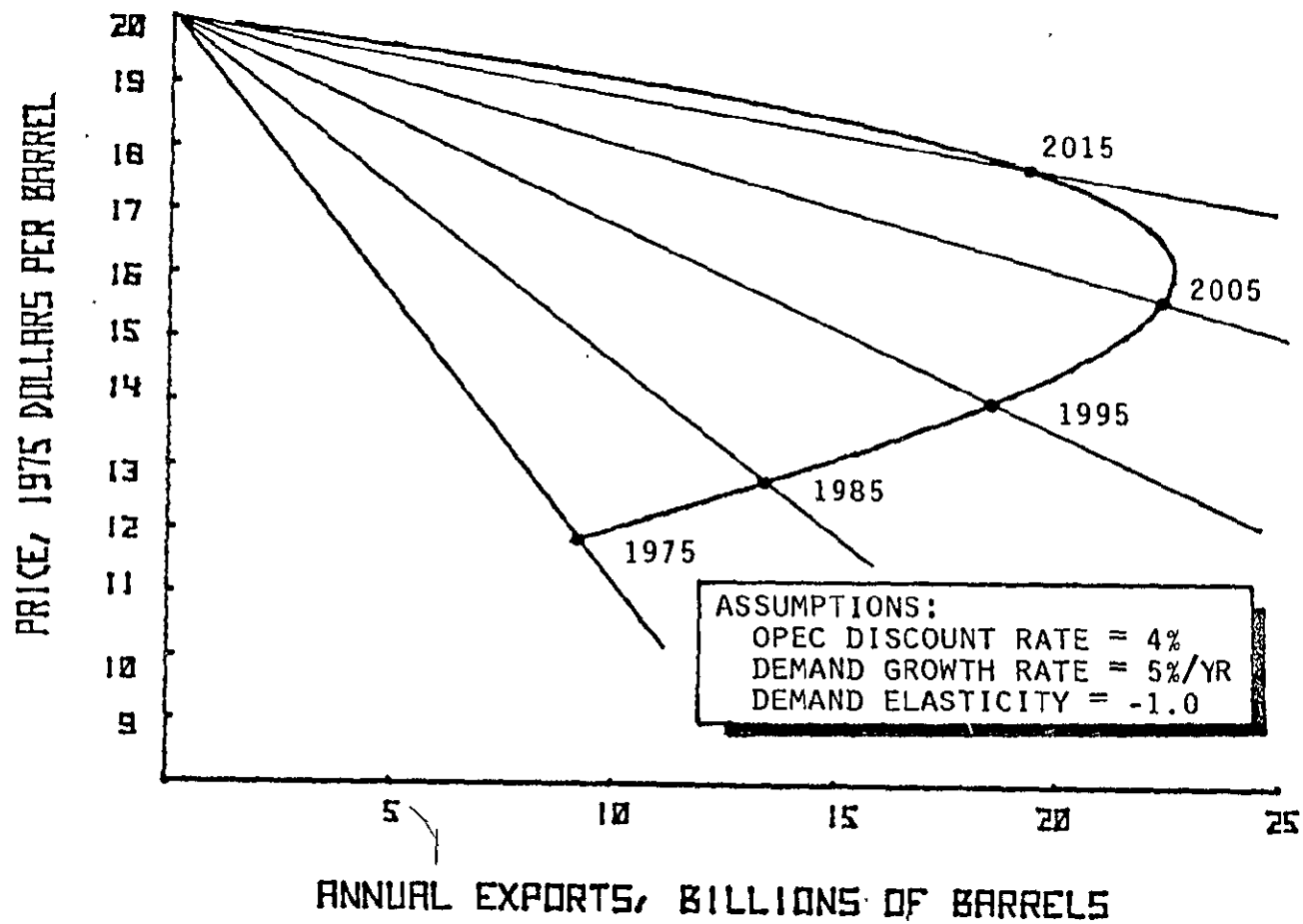


Figure 2.4 An Optimal Price-Quantity Trajectory

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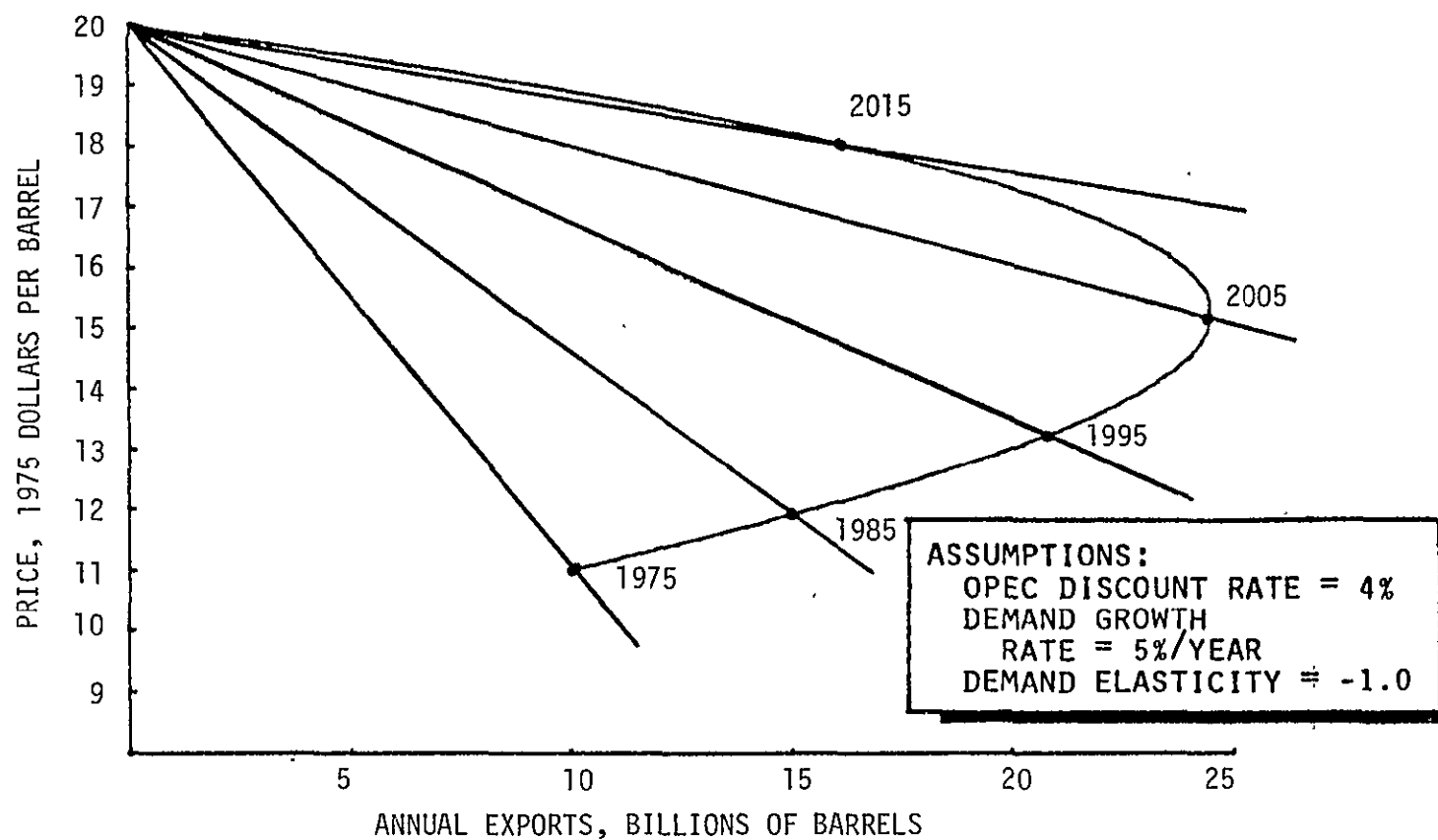


Figure 2.5 An Optimal Price-Quantity Trajectory With An Expected Alternative Arrival Time Of 50 Years

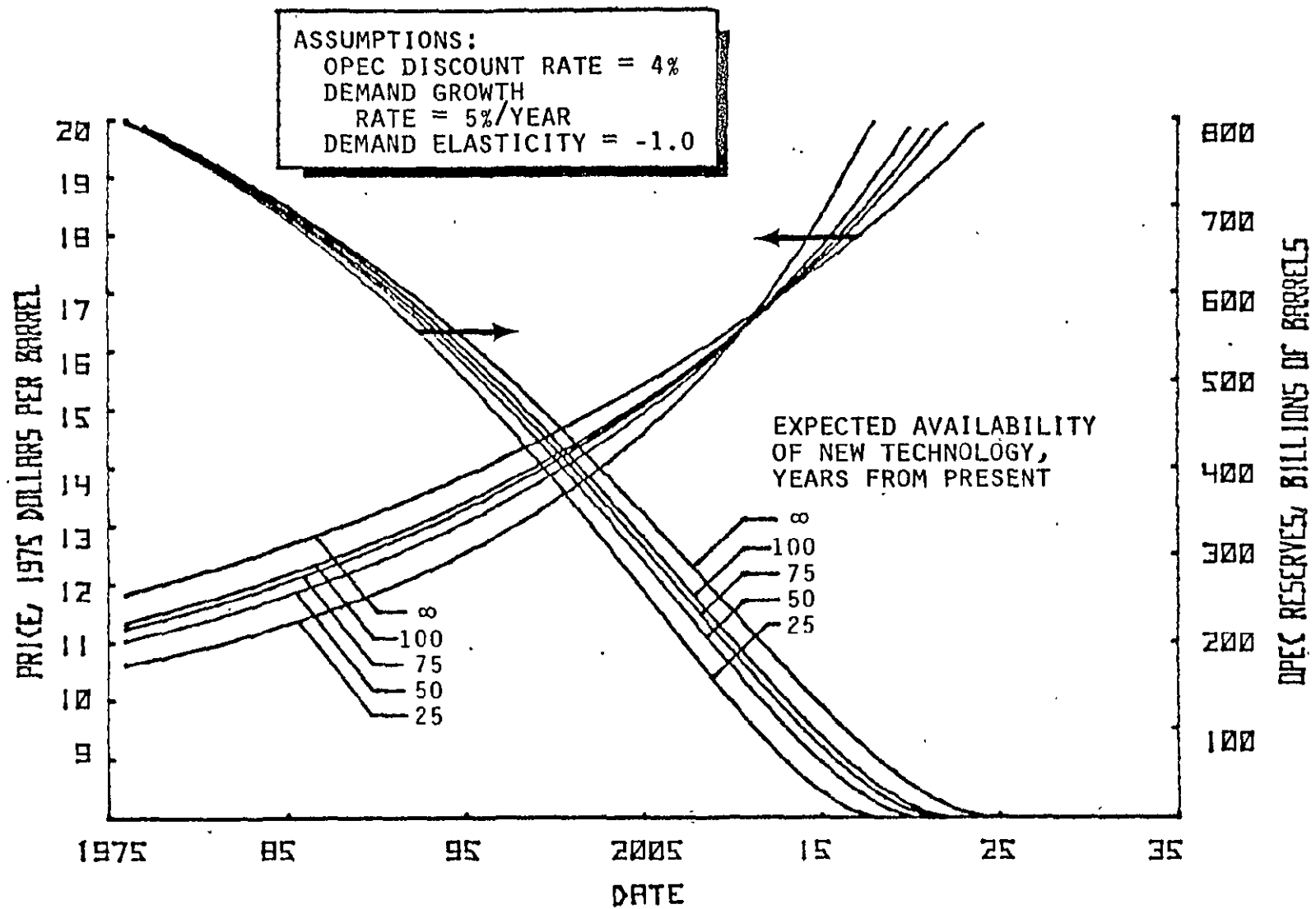


Figure 2.6 Optimal Consumption Patterns With Energy R&D

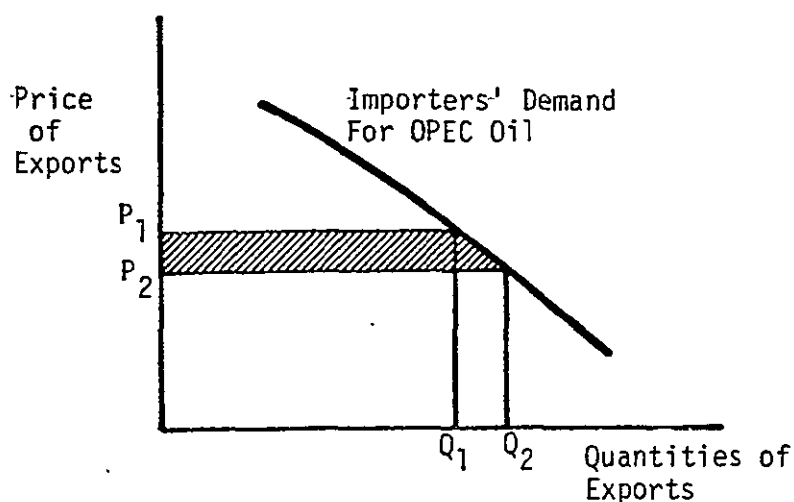


Figure 2.7 Benefits to Importers From Export Price Reduction

yields the total expected benefit. From the time of substitute availability on, the price path such as shown in Figure 2.6 no longer applies because the substitute would then be produced. The probability of the substitute not yet being available is determined by  $\omega_t$ ; it starts at unity and drops off exponentially over time. Thus, the time periods where the "with" price exceeds the "without" price are given very little weight.

The present value of benefits, due to the effect of energy R&D on fossil fuel consumption patterns as described above, assuming three OPEC rates of discount, are presented in Figures 2.8, 2.9 and 2.10.

## 2.5 Potential Benefit Attributable to SPS

The potential fossil fuel price effect benefit due to an SPS development program is extremely difficult to analyze. It depends upon many policy decisions, upon environmental factors such as  $\text{CO}_2$  in the atmosphere, and upon the numerous other potential long-range energy alternatives. It also requires an assessment of the probability of success in each potential energy program and of the resulting impact on the demand for OPEC crude oil

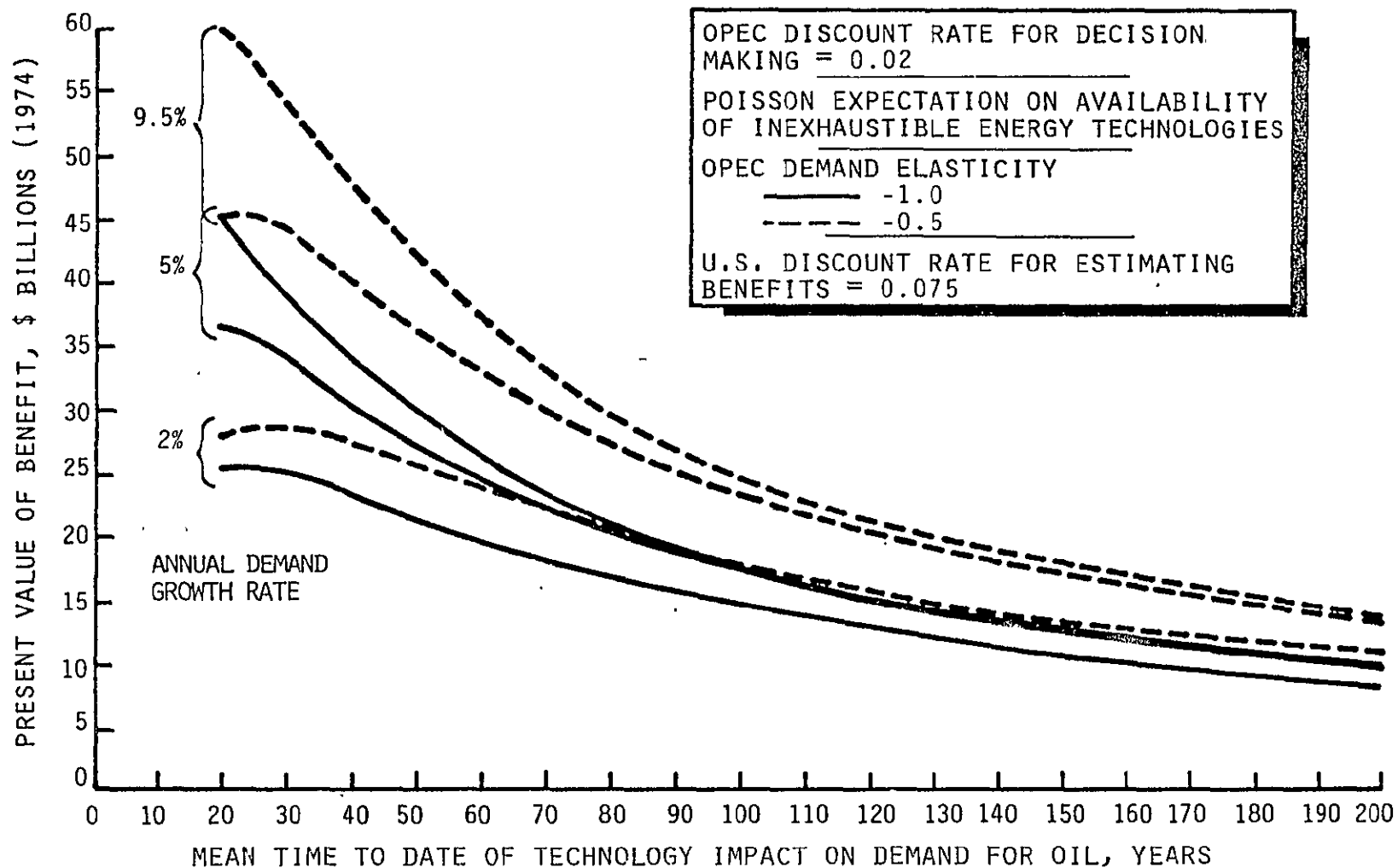


Figure 2.8 U.S. Benefits Of Energy R&D Due To Price Change On Oil Imports From OPEC--2% CASE

1993

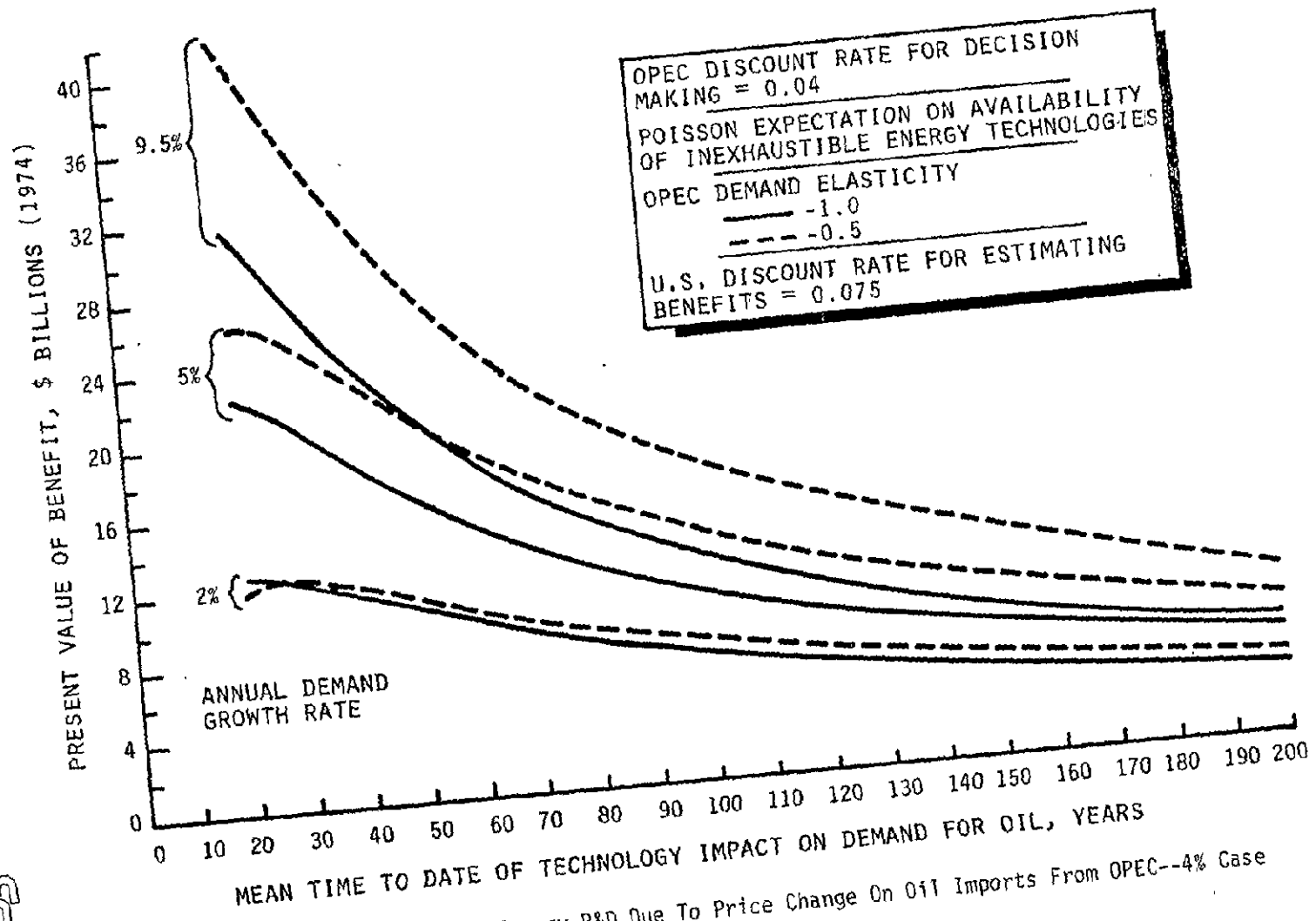


Figure 2.9 U.S. Benefits Of Energy R&D Due To Price Change On Oil Imports From OPEC--4% Case



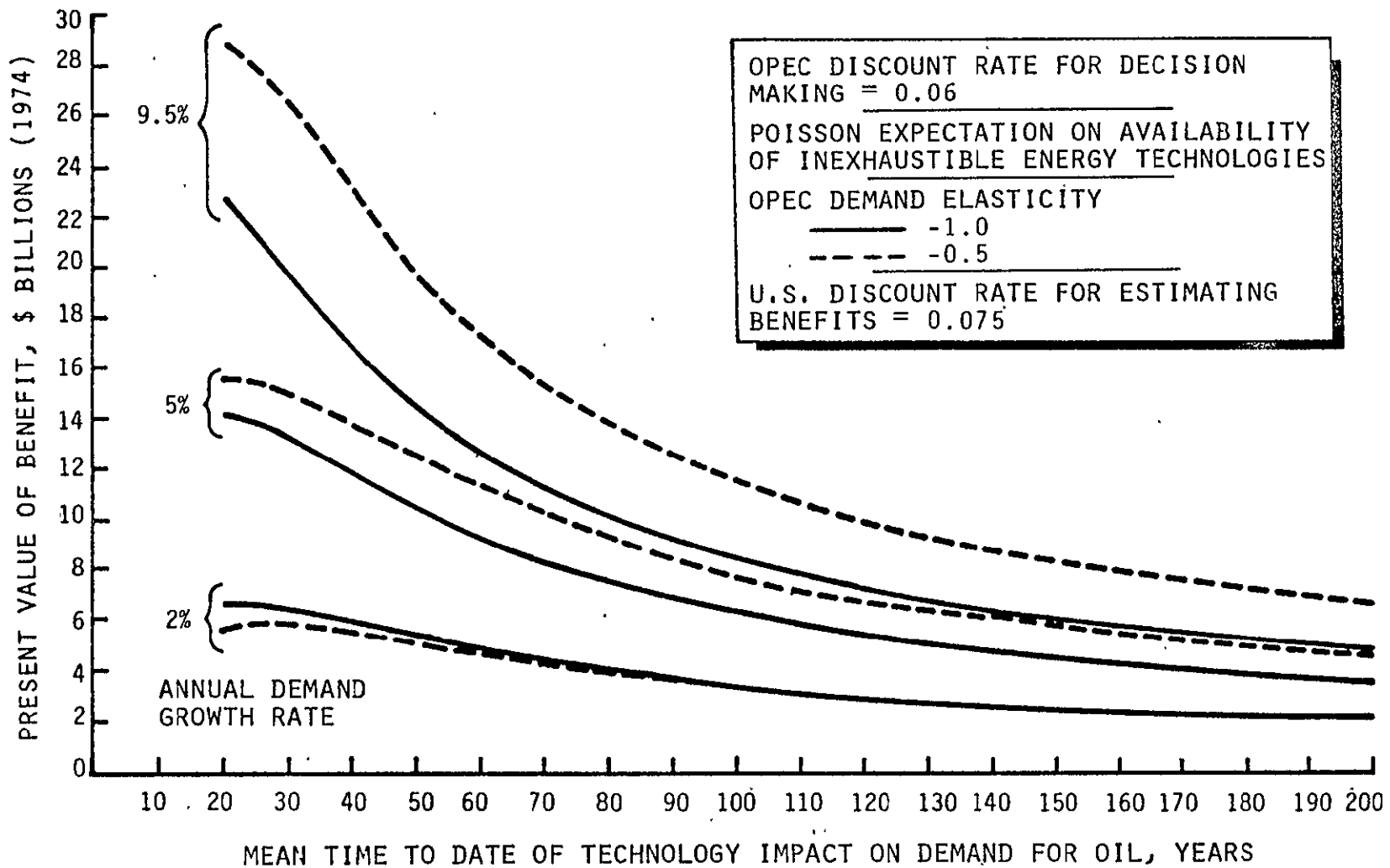


Figure 2.10 U.S. Benefits Of Energy R&D Due To Price Change On Oil Imports From OPEC--6% Case

as a function of time. In this context, the effort presented here is extremely preliminary and yields only very rough order-of-magnitude benefit estimates.

It is assumed below that only non-fossil-fuel energy sources will reduce the demand for OPEC crude and that these sources will be either fusion, breeder reactor or SPS. The probabilities of first commercial plant, as a function of time, for these alternatives is assessed in Figures 2.11, 2.12 and 2.13. The assessment for fusion derives directly from the work of Vanston et al. for a fusion R&D program at a moderately increased funding level over the Department of Energy baseline plan.<sup>\*</sup> The assessment for breeder reactor technology is strictly ad hoc. It is based on an earliest availability date of 1985<sup>\*\*</sup> with a probability of commercialization rising to 90 percent over 20 years. Finally, the assessment for SPS is based partly on the previous work of ECON,<sup>\*\*\*</sup> which indicates approximately an initial probability of economic and technical success of 30 percent, and partly on the ad hoc assumptions that this probability increases to 80 percent over 25 years and that there is a constant 50 percent chance of social and environmental acceptability given technical and economic success.

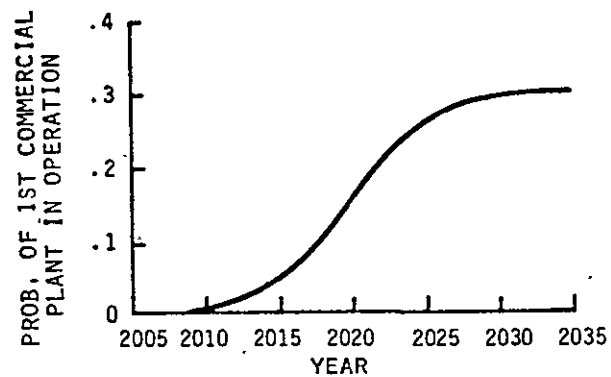
Next, the assessment of market penetration for these three technologies is presented in Figures 2.14 and 2.15. The market penetration for fusion was obtained from the Brookhaven National Laboratory. It is the

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<sup>\*</sup> Fusion Power by Magnetic Confinement Program Plan, ERDA-76/110/2, July 1976.

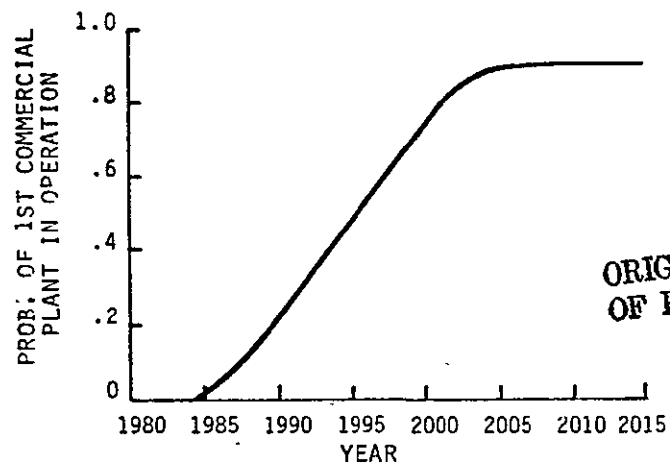
<sup>\*\*</sup> The LMFBR--Its Need and Timing, ERDA-38, May 20, 1975.

<sup>\*\*\*</sup> Space-Based Solar Power Conversion and Delivery Systems Study, Final Report, ECON, Inc. Report No. 77-145-1, March 31, 1977.



SOURCE:  
J.H. VANSTON, S.P. NICHOLS,  
R.M. SOLAND, PAF--A NEW PROB-  
ABILISTIC, COMPUTER-BASED  
TECHNIQUE FOR TECHNOLOGY  
FORECASTING, UNIV. OF TEXAS  
AT AUSTIN, 1976. (WORK  
PERFORMED FOR DIVISION OF  
MAGNETIC FUSION ENERGY)

Figure 2.11 Probability of Implementation of Fusion  
Given Moderately Increased Funding



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ASSUMES: EARLIEST AVAILABILITY OF  
BREEDER IS ABOUT 1985 --  
PROBABILITY OF TECHNICAL  
AND ECONOMIC SUCCESS  
BECOMING 90% IN 2005 --  
ENVIRONMENTAL AND SOCIAL  
ACCEPTABILITY NOT ASSESSED

Figure 2.12 Probability of Implementation of  
Breeder Reactor Technology

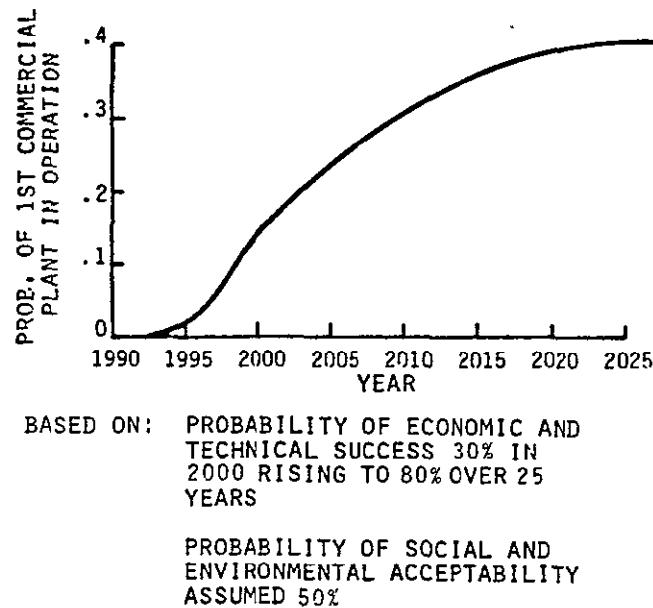


Figure 2.13 Probability of Implementation of SPS

currently used model for economic assessment of fusion. Contrary to ERDA-38, which estimates exceedingly high implementation rates for breeder reactor technology (up to 125 GWe/year), the implementation model for breeder technology is assumed to be the same as the implementation model for fusion. The rationale for this assumption lies in the similarities of the two technologies: both are ground based, both about the same magnitude, both potentially about the same cost. The implementation model for SPS was taken from the current Marshall Space Flight Center SPS implementation scenario.

Using the data of Figures 2.11 through 2.15, the expected value (mathematical expectation) of market penetration for each individual technology was computed as a function of time. Also, assuming independence between technologies, the expected market penetration resulting from R&D programs combining the candidate technologies was computed. These data

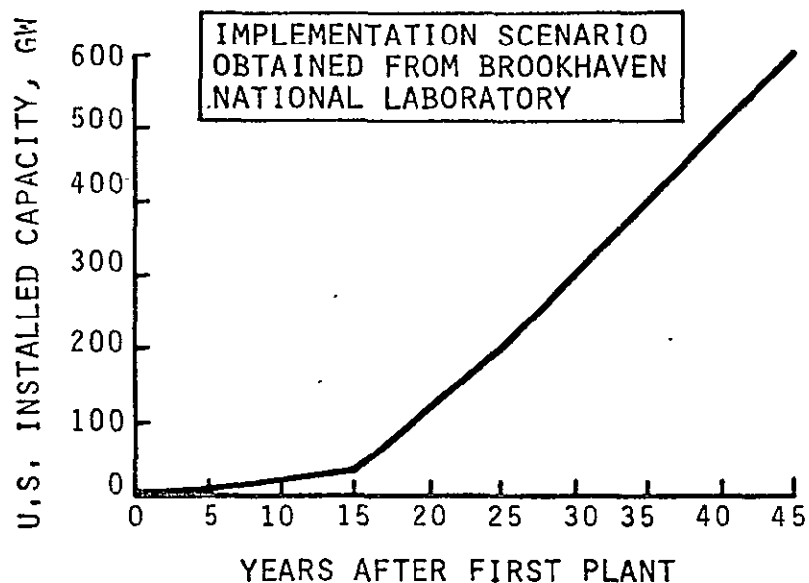


Figure 2.14 Implementation Scenario for Fusion and Breeder Reactor

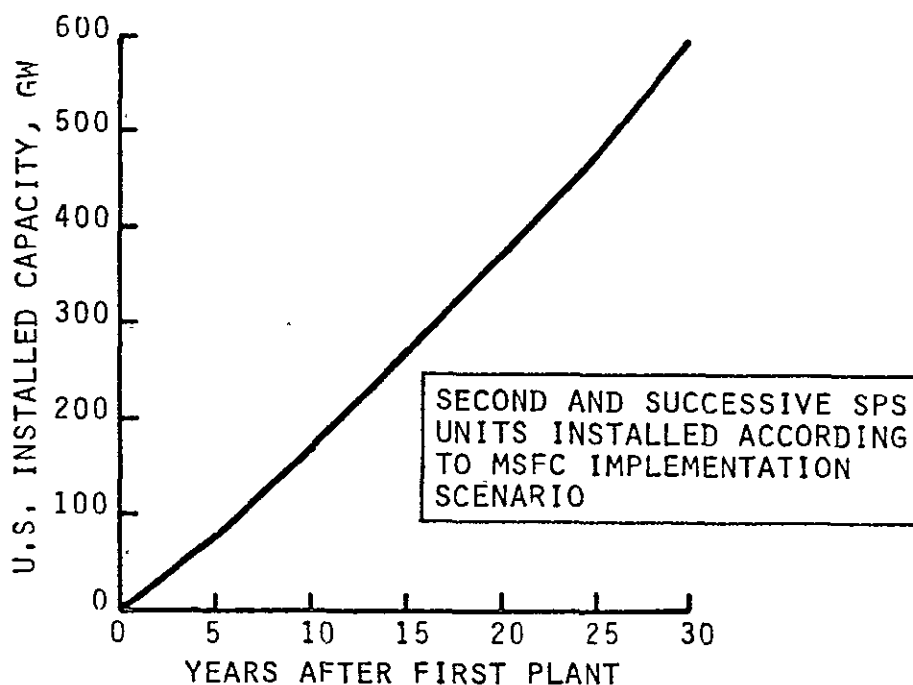


Figure 2.15 Implementation Scenario for SPS

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are shown in Figure 2.16. Note that the expected value of installed capacity is needed due to the probabilistic nature of the oil price model.

Finally, the Poisson parameter is picked off of Figure 2.16 as follows: It is taken as the time difference, in years, between 1977 and the date at which the expected value of installed capacity reaches 600 GWe divided by 0.6931. This division provides an estimate of the Poisson parameter, or e-folding time, in terms of the 50 percent cumulative probability point. This time is then used as the mean time to date of technology impact on demand for oil as given in Figures 2.8, 2.9 and 2.10. The time at which 600 GWe of expected installed capacity was chosen to determine the value of the Poisson parameter because 600 GWe represents a very substantial level of installed capacity--it is about equal to the current U.S. installed generation capacity. Given a modest growth in total U.S. electric energy demand, this capacity would be such as to have, possibly, even a negative impact on the demand for coal, thereby freeing this resource for conversion to liquid and gaseous fuels.

The above procedure is admittedly ad hoc and extremely crude. However, it serves to illustrate, first, that the total benefit must be divided across a number of potential new technologies and, second, that the procedure for estimating the benefits attributable to any one particular technology is quite cumbersome and subject to a number of very flexible inputs. With this qualification in mind, Table 2.1 gives some very rough order-of-magnitude benefit estimates. From this table, one easy conclusion can be drawn: If the breeder reactor is widely accepted, it would likely be the main technology to capture the fossil fuel price effect benefit. If breeder reactor implementation is disallowed on political, social or

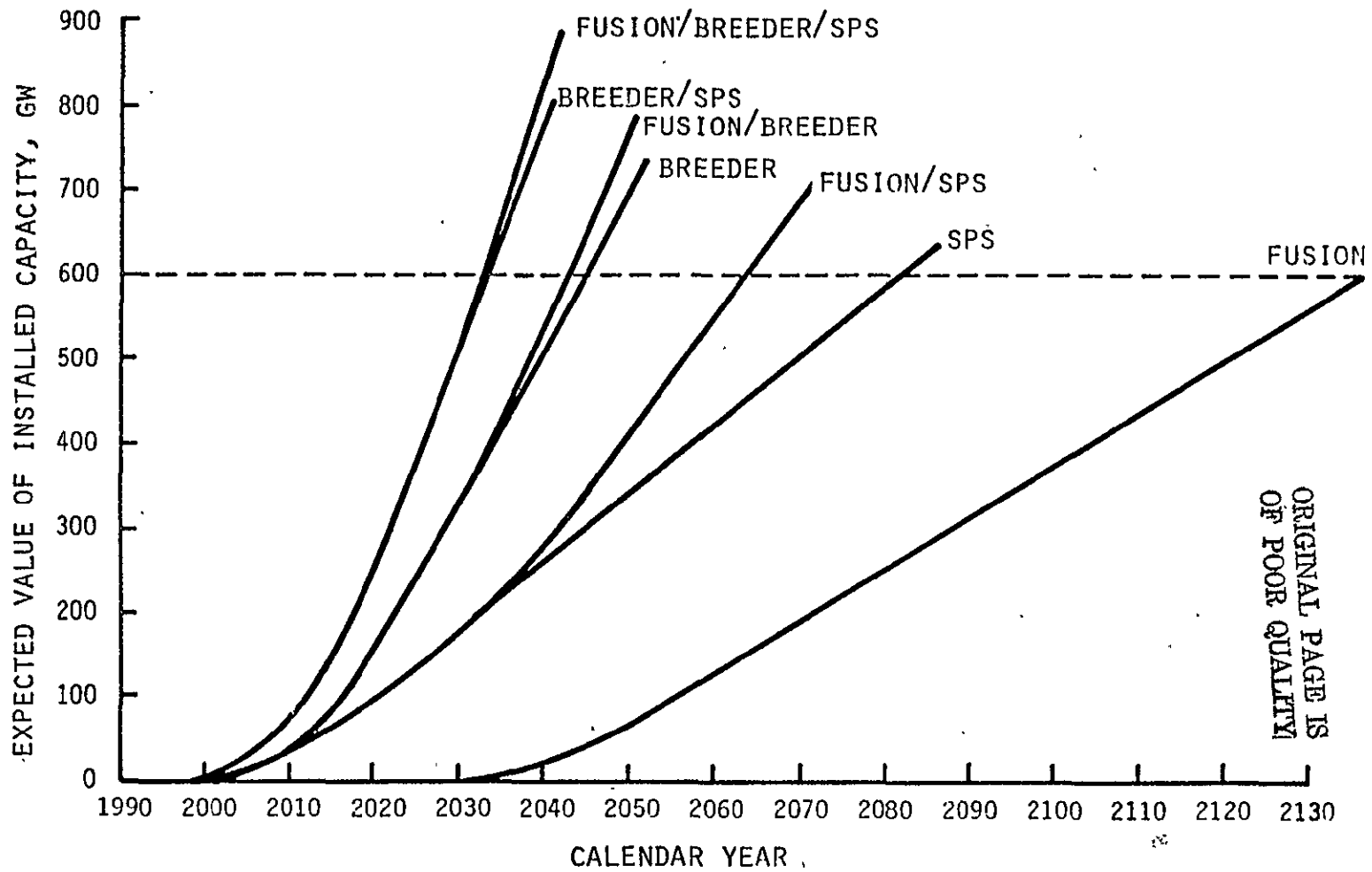


Figure 2.16 Expected Value Market Penetration Model

TABLE 2.1 VERY ROUGH ORDER-OF-MAGNITUDE BENEFIT ESTIMATES

| TECHNOLOGY<br>ALTERNATIVE                                                                                                                                                                                                                                                                                                                                                      | PRESENT VALUE<br>OF DEVELOPMENT<br>COST, \$B | PRESENT VALUE<br>OF GROSS<br>BENEFIT, \$B | PRESENT VALUE<br>OF NET<br>BENEFIT, \$B |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------|-------------------------------------------|-----------------------------------------|
| FUSION                                                                                                                                                                                                                                                                                                                                                                         | 2-7                                          | 5.5                                       | -1.5 to 3.5                             |
| BREEDER                                                                                                                                                                                                                                                                                                                                                                        | 5                                            | 10.2                                      | 5.2                                     |
| SPS                                                                                                                                                                                                                                                                                                                                                                            | 12                                           | 7.2                                       | -4.8                                    |
| FUSION/SPS                                                                                                                                                                                                                                                                                                                                                                     | 14-19                                        | 8.5                                       | -10.5 to -5.5                           |
| BREEDER/SPS                                                                                                                                                                                                                                                                                                                                                                    | 17                                           | 12.0                                      | -5.0                                    |
| FUSION/BREEDER                                                                                                                                                                                                                                                                                                                                                                 | 7-12                                         | 10.6                                      | -1.4 to 3.6                             |
| FUSION/BREEDER/SPS                                                                                                                                                                                                                                                                                                                                                             | 19-24                                        | 12.0                                      | -12.0 to -7.0                           |
| <p>ASSUMPTIONS</p> <ul style="list-style-type: none"> <li>● BASED ON 4% OPEC DECISION RATE OF DISCOUNT, 5% ANNUAL OPEC DEMAND GROWTH, -1.0 OPEC DEMAND ELASTICITY</li> <li>● NET BENEFITS CITED ARE STRICTLY ILLUSTRATIVE AND SHOULD NOT BE USED TO ESTABLISH ADVOCACY</li> <li>● ONLY FOSSIL FUEL PRICE EFFECT BENEFITS ON OPEC IMPORTS TO U.S. ARE TREATED ABOVE.</li> </ul> |                                              |                                           |                                         |



environmental grounds, however, other long-range energy alternatives take on a far greater importance. Last, while the development program cost estimates are also very crude (and possibly even unlikely), it is appropriate to point out that SPS is high compared to breeder reactor and fusion. Therefore, to improve the position of SPS in a comparative evaluation against alternatives, a key element would be to reduce the SPS development cost.

### 3. A STUDY OF BENEFITS ATTRIBUTABLE TO ALTERNATIVE USES OF SPS TECHNOLOGIES

The purpose of the study reported in this section is to identify SPS technologies which might find markets elsewhere. In theory at least, if these technologies have value outside of the SPS program, other programs and areas which receive these benefits should be willing, to some extent, to provide funding, or at least advocacy, for the development of these technologies. First, a number of potential alternative applications of SPS technologies are identified and an overview given concerning the types of benefits which might result. Then four technologies which appear to offer the greatest benefits from alternative applications are discussed in more detail. Finally, conclusions and recommendations for further study are made.

#### 3.1 Overview of Potential Alternative Applications

SPS relies on a large number of economically unproven technologies such as solar cell conversion, microwave power transmission, the utilization of composite structural materials, and space transportation and assembly. Whereas the untested nature of these technologies expands the range of possible benefits from alternative applications, the uncertainty involved makes estimates of markets and corresponding benefits difficult. Table 3.1 summarizes the technologies utilized by SPS which are thought to have large potential benefits from alternative applications and the areas in which these benefits might be realized. In the area of construction techniques, two characteristics which distinguish SPS from other types of construction are the need to fabricate very large structures and the use

Table 3.1 SPS Technologies With Possible Alternative Applications

- CONSTRUCTION
  - LARGE-SCALE ASSEMBLY TECHNIQUES
  - TELEOPERATORS
- MICROWAVE TRANSMISSION
  - TERRESTRIAL POWER TRANSMISSION AND HIGH-ALTITUDE POWERED PLATFORMS
  - INDUSTRIAL APPLICATION MICROWAVE RADIATION
  - CONSUMER GOODS (E.G., MICROWAVE OVENS)
  - PHASE CONTROL TECHNOLOGY (COMMUNICATIONS)
- MATERIALS
  - REFLECTORS
  - COMPOSITE MATERIAL TECHNIQUES AND APPLICATIONS
  - CRYOGENICS (STORAGE AND HANDLING TECHNOLOGIES AS WELL AS  $\text{LH}_2$  MFG.)
  - SOLID STATE APPLICATIONS OF SOLAR CELL MATERIAL ( $\text{Si}$ ,  $\text{GaAs}$ , OR  $\text{CdS}$ )
- POWER DISTRIBUTION AND SWITCHING
  - UTILITIES
  - INDUSTRIAL APPLICATIONS
- SPACE TRANSPORTATION
  - TOURISM
  - OTHER LARGE STRUCTURES

of remote-controlled devices to assist in assembly. Other construction techniques such as joints and fasteners may have alternative application, but not enough is known about how SPS will actually be constructed to estimate benefits in other areas. The technology employed in teleoperators might find application in the handling of radioactive waste or other biologically hazardous materials, and reprocessing of nuclear fuels, or military applications such as the handling of explosives or rocket fuel. Submersible technology might employ some of the advances brought about by SPS research, as might also the development in pursuit of undersea oil and mineral extraction or scientific exploration. To the extent that they prove cost effective, remote systems technology might be employed to assist or replace man in any hazardous occupation or environment, whether it is coal mining, the decommissioning of nuclear power plants, the rebuilding

of fusion power plant inner walls or operations in arctic regions.

Finally, remote systems technology could find application in space manufacturing or planetary exploration.

Microwave power transmission systems similar to that which might be used for SPS offer the possibility of terrestrial power transmission through closed waveguides or the powering of high-altitude platforms (such as dirigibles or drone aircraft operating at about 70,000 feet) to be used for communications broadcast or relay, forest fire or coastal surveillance or other remote-sensing purposes. It should be noted that these are the only currently conceived applications of the entire microwave power transmission system. In addition, individual components of the system such as the amplitrons and waveguides might be employed in industrial processes, principally for heating and drying in such applications as the drying of materials in foundries or rubber products manufacturing plants or the tempering of meat. Microwave radiation might be attractive in any situation where rapid or uniform heating is desirable. The primary example of the utilization of an amplatron-like device in the production of a consumer good is, of course, the microwave oven which employs a magnetron. Finally, there are potential applications of developments in phase control technology prompted by SPS for communications purposes.

Among the untested technologies on which SPS might rely are a number of materials which have not been used yet in broad-scale application such as the extremely lightweight reflecting material for concentrating sunlight on the solar cell panels, the solar cells themselves, and composite materials from which the structure might be derived. The reflectors might

be used in terrestrial solar thermal systems, and the solar cells have already found a number of applications as small-scale remote power sources (for example, in powering buoys, microwave repeaters, highway call boxes), in addition to their widely recognized use in powering satellites. Numerous other applications of solar cells will be possible as the price drops over time, a particularly large market being offered by on-site residential and commercial power generation when solar array installed system prices reach the 1-2  $\$/W_p$  (peak watt) range. If solar cells were to become even less expensive (less than  $\$0.50/W_p$ ), they would likely be economically competitive with other power sources for central power generation. A very substantial potential market exists in the electrification of rural areas in less-developed countries at prices higher than those required for price competitiveness in the domestic market. Furthermore, the technology which would be required for producing large amounts of very pure solar cell material (typically, gallium arsenide, silicon, or cadmium sulfide) might make possible other semiconductor applications of whatever material is chosen for use in SPS. Finally, the space transportation and the positioning and attitude control scenarios which are currently envisioned for SPS would necessitate the manufacturing and handling of large amounts of cryogenic materials (liquid oxygen and liquid hydrogen). The principal alternative application of the manufacturing, storage and handling technologies which would evolve might arise in the support of broader application of superconducting technologies such as the superconducting magnets which would be required for plasma fusion reactors or superconducting cables for low-loss, long-distance power transmission.

SPS will require power distribution and switching technologies capable of handling extremely large power levels and capable of dealing with solar panel destruction or failure (whether caused by micrometeorite collision, material failure or degradation) in such a way as to prevent further damage. The knowledge gained in solving these problems might find applications both in utilities and in industrial settings. However, it is difficult at this point to specify what the nature of SPS advances might be and, hence, what the corresponding benefits might be from improvements on current technology. More technical detail will be required before benefits may be assessed in this area.

Finally, the economic feasibility of SPS depends in part on the development of a low-cost space transportation system. In general terms, anything that is currently done in space, or anything being considered, will become more attractive if the cost of space transportation (which is presently a significant portion of the total cost of most space activities) is reduced. However, the only other projects which would require the development of heavy lift launch vehicles are the construction of other large structures in space or the development of tourism.

### 3.2 The Nature of Potential Benefits

Benefits from alternative applications of SPS-derived or developed technologies may occur either from the creation of new products or from the improvement of existing products. Benefits of the first type are generally more difficult to estimate in that the market demand for a new product is uncertain. The further in the future the time frame in which one is trying to predict a market demand, of course, the greater the uncertainty. To the extent that a market demand may be estimated, the

standard measure of benefit is what is known as "consumers' surplus," which is the total amount that purchasers of a given commodity would have been willing to pay in order to obtain it, over and above the prevailing market price ( $P_0$ ). This consumers' surplus is represented as the shaded area under the demand curve in Figure 3.1 and above the prevailing market price. Consequently, the benefit of developing a new product would be the entire consumers' surplus associated with the market for that product.

The consumers' surplus technique may also be applied in estimating benefits of the second type mentioned above, that is, the improvement of an existing product or capability. If a technological improvement causes a reduction in the price of a product, say from  $P_1$  to  $P_0$  in Figure 3.1, the benefit associated with this price drop is the shaded area between the two respective price levels. Such a price drop will usually reflect a decrease

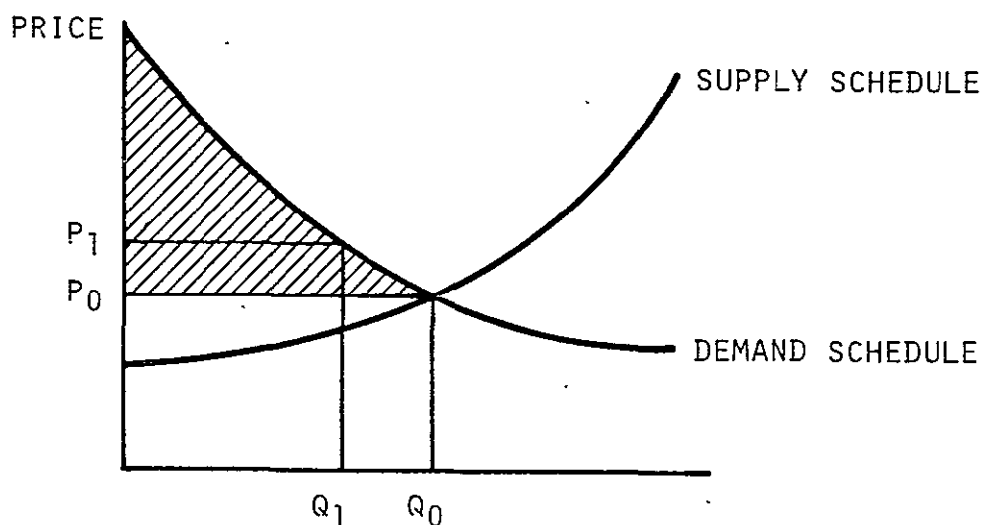


Figure 3.1 Portrayal of Consumers' Surplus  
as a Means of Estimated Benefits  
Associated with a Particular Product

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in the manufacturing cost of a product. Other types of product improvement include reduction in operation or maintenance costs or greater material or energy utilization efficiency, in which case the benefit may be estimated directly from the personnel, energy or material cost saving.

Estimates of benefits in this study, where they were possible, are limited to the near- to mid-term (through the year 2000) and are strictly order-of-magnitude, the purpose of these estimates being only to help order the significance of different areas of potential benefits.

### 3.3 SPS Technologies Selected for Study and Their Potential Benefits

From the list of potential alternative applications described in Section 3.1, four areas were selected for study based on the size of the apparent potential benefit, the existence of an identifiable product and/or market, and the availability of information on the product or market. These technologies are discussed below in the order of the significance of their potential alternative applications.

#### 3.3.1 Solar Cell Technology

There are some current applications in which solar cells, while relatively expensive, are nonetheless more desirable than alternative power sources. These range from an extreme represented by satellite power systems, where costs of thousands of dollars per kilowatt have been deemed acceptable, to power sources for remote locations like buoys, where system costs have typically been in the \$100-\$200 range. As the price drops from these levels other applications develop, as described in Section 3.1. The



current solar cell market has been characterized<sup>\*</sup> as being fairly small, having installed 400 kW<sub>p</sub> (kilowatts-peak) of array capacity in the non-communist world in 1976, with high unit system costs in the 15-40 \$/W<sub>p</sub> range, depending upon the nature of the application. This is the typical price range for such applications as power sources for microwave repeaters.

The estimates of the annual market for solar cells at different cost levels for installed systems shown in Figure 3.2 were derived from data developed by The BDM Corporation and Solarex Corporation.<sup>\*\*</sup> They conducted an analysis of present applications and projected markets for these applications under a variety of assumptions concerning market potential: that market potential is based on life cycle cost over a 20-year time horizon, that it is based on first cost only, and that the market potential is 10 percent of the life-cycle cost-effective market. These estimates of the market are for 150 applications "selected as the most promising from among over 1000 applications that were identified in the course of a study for ERDA." Note that the market curves do not show the potential for system prices less than \$0.50/W<sub>p</sub> which would be the point at which large-scale commercial and utility applications might become competitive. BDM and Solarex projected the market for the present applications through the year 1986, and their forecast is shown in Figure 3.2, with an extension of this forecast representing a potential cost trend.

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<sup>\*</sup>"An Analysis of the Potential Industry/Market Impacts of the Federal Photovoltaic Utilization Program (FPUP)," by The BDM Corporation, submitted to the Department of Energy, (27 October 1977).

<sup>\*\*</sup>"Photovoltaic Power Systems Market Identification and Analysis," Task 1 Report prepared by The BDM Corporation and Solarex Corporation for ERDA, May 1977.

- ASSUMPTIONS:**
- POTENTIAL MARKET = 10% OF THE COST-EFFECTIVE MARKET
  - MARKET PENETRATION: 18% IN 1978, 50% IN 1985, 90% IN 1995
  - MARKET INCLUDES ONLY PRESENT APPLICATIONS
  - SOLAR CELL COST = ONE-HALF OF INSTALLED SYSTEM COST

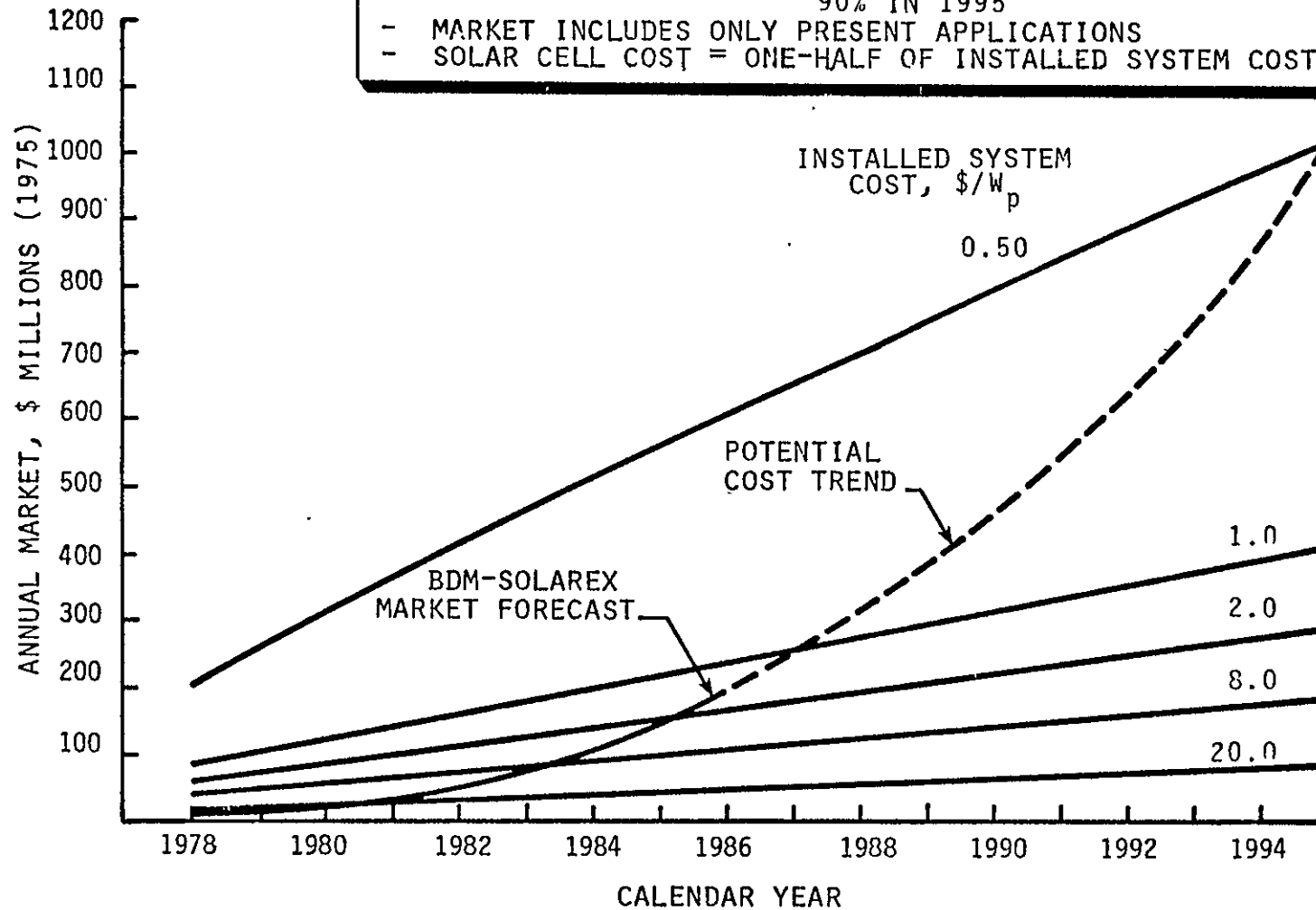


Figure 3.2 Market for Solar Cells as a Function of Installed System Price

The projections of market demand at any given price, assuming the potential market to be 10 percent of the cost-effective market, were subjected to the assumptions by BDM in its baseline case for the FPUP study concerning market penetration (namely, an 18 percent penetration in 1978, 50 percent by 1985 and 90 percent by 1995), to produce the market curves shown in Figure 3.2. As these curves reflect only the market for present applications and do not assume any government intervention or support, they may seem somewhat conservative.

The market demand scenarios were converted to estimates of annual benefits as a function of system cost, according to the consumers' surplus methodology described in Section 3.2. The potential benefits are presented for three different points in time (1978, 1985 and 1995) in Figure 3.3. It can be seen that the potential benefits for reducing the system cost below 3-4  $\$/W_p$  are quite large, especially at higher levels of market penetration, such as assumed for 1995. The difference between two levels of benefits corresponding to two different cost levels is a measure of the benefit attributable to SPS or any other project which is responsible for the cost reduction.

It should be noted that there are differences between the requirements for solar cells to be used in space and those to be used on the ground, as well as a difference in the relative importance of different solar cell characteristics. Space-based solar cells must be resistant to high-energy proton and electron fluences and must be ultra lightweight in order to reduce transportation cost; neither of these factors is important in terrestrial utilization. Furthermore, higher concentration ratios

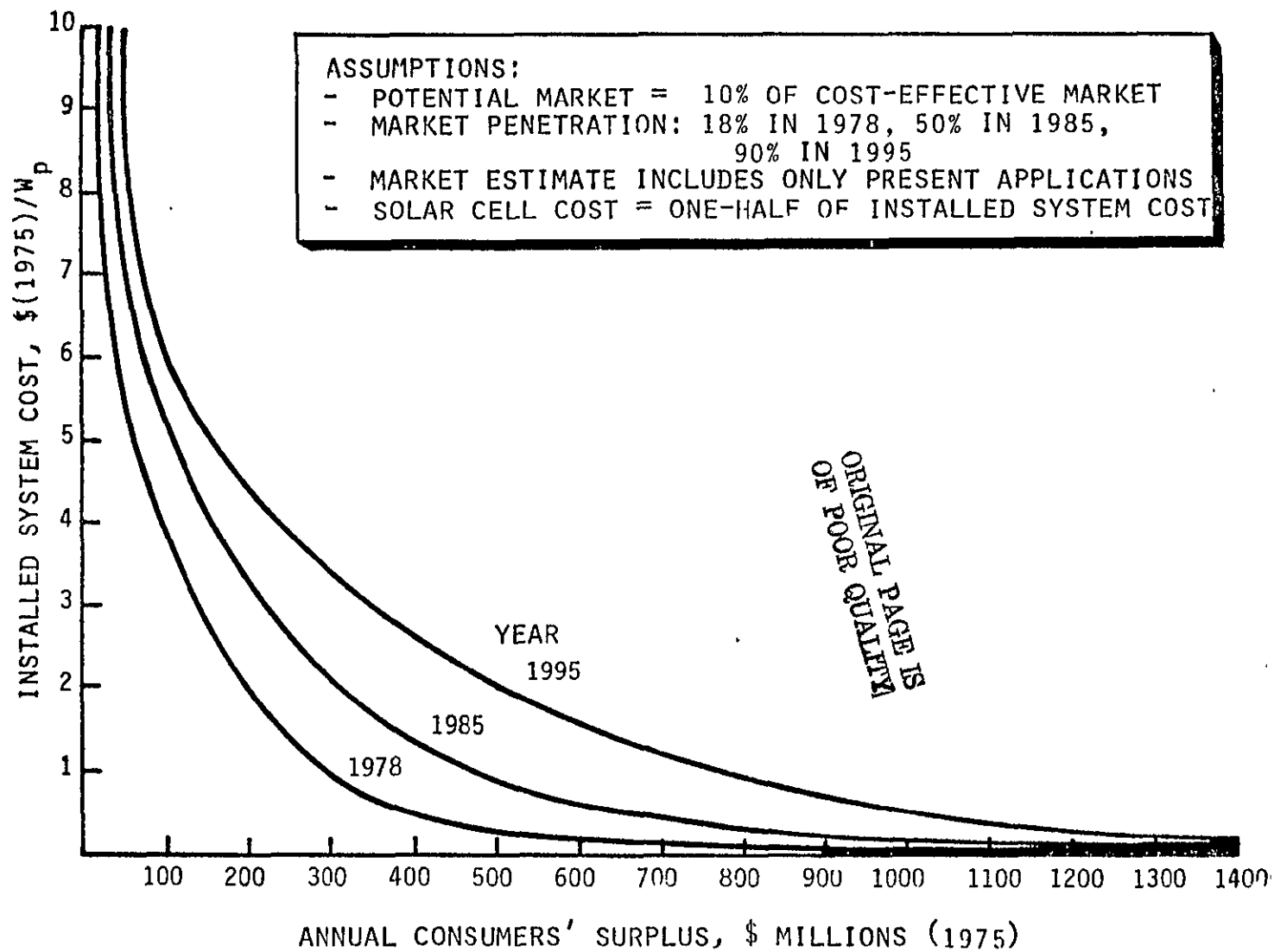


Figure 3.3 Potential Annual Benefits in the Solar Cell Market Versus System Price

might be more practicable in space than they will be on the ground, hence, SPS might require a solar cell material such as gallium arsenide, which is susceptible to high concentration with lower conversion efficiency losses resulting from the higher temperature than other materials such as cadmium sulfide or silicon. A variety of different materials and methods for solar cell manufacture are currently being experimented with, and surely more will develop in the future. In order to select the appropriate material for SPS, trade-off studies will have to be conducted accounting for cell mass, efficiency, cost, and susceptibility to radiation and concentration. In order for benefits to accrue to SPS, SPS-related development would have to reduce the cost or improve the performance of solar cells appropriate for terrestrial use. At this point it is not clear whether the cell which is developed for terrestrial use will drive the SPS selection, or whether the reverse should be true, or whether the two uses should (or will) be totally independent.

### 3.3.2 Microwave Transmission Technology

Terrestrial power transmission by microwaves has been considered as a potentially significant alternative application of the microwave power transmission system (MPTS). Power transmission over various distances up to 1800 km, using different types of closed waveguides, was examined,<sup>\*</sup> but was not found to be economically attractive compared to most alternative forms of power transmission.<sup>\*\*</sup> The closed waveguides bring about very

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<sup>\*</sup> Monthly Progress Report by the Arthur D. Little Co. to ECON, Inc., re terrestrial power transmission, April 1975.

<sup>\*\*</sup> "Space-Based Solar Power Conversion and Delivery Systems Study," Interim Summary Report, prepared for NASA-MSFC, March 31, 1976.

high costs, compounded by the requirement that they be very precisely aligned in order to prevent transmission losses. Consequently, microwave technology does not appear to be a candidate at this time for the replacement of conventional means of utility power transmission.

One application of the MPTS--the only application which would involve the entire system--in which it would apparently be cost effective<sup>\*</sup> is the powering of high-altitude platforms which could be used for various remote-sensing purposes such as forest fire or coastal surveillance, or for communications broadcast or relay. These platforms, whether in the form of rigid airships or drone aircraft, would operate at an altitude of about 20 km. The use of microwave transmission to power such platforms was found likely to be cost competitive with other means, such as the rotation of fossil-fuel-driven drone aircraft. Because transmission of power would be straight up, the problems which occur with maintaining an acceptable power-density level for individuals in the immediate area appear to be manageable. Batelle Memorial Institute has studied<sup>\*\*</sup> a number of different payloads and missions for such high altitude platforms. The benefits corresponding to this use of the MPTS would be the cost savings achieved by powering these platforms using microwave transmission, as opposed to alternative methods. From the standpoint of SPS planners, such an application might be desirable in that it would lead to public awareness of and

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<sup>\*</sup>James W. Sinko, "High Altitude Powered Platform Cost and Feasibility Study," prepared for NASA Headquarters, October 12, 1977 (draft report).

<sup>\*\*</sup>(Awaiting a copy of the report)

experience with the type of system designed to transmit power to Earth from an SPS.

There is currently about a \$20 million per year market for industrial heating applications of microwave power. This typically amounts to about 40 units per year rated at 50-75 kW for specialized uses such as tempering meat, drying in foundries (where microwave devices replace gas drying ovens), and heating in rubber factories (where microwave devices are useful because they are able to heat materials rapidly and uniformly). This is a market which is beginning to expand and is likely to expand more rapidly in the future as the price of natural gas (which is usually used for such heating and drying functions) rises with respect to the price of electricity. To the extent that SPS-derived technological advances reduce the cost of producing amplitrons and related equipment, and correspondingly expand the market for products using these devices, a benefit may be attributed to SPS. However, a more extensive analysis of the nature of technological improvements and likely cost reductions offered by SPS, as well as the potential market for industrial heating devices, will be required before the resulting benefits can be estimated. Amplitrons prove to be superior to magnetrons in use with serpentine waveguides for sheet of film application of microwave energy. However, there is not currently a very large market for such devices, only about 2000 units per year within the next few years. From the standpoint of developing SPS technology this represents only a "first step on the learning curve."

A familiar product which takes advantage of technology similar to that employed in the MPTS is the microwave oven, which uses a single-envelope, single-bottle magnetron similar to the amplatron to be utilized

in SPS (except that the SPS amplatron is slightly more sophisticated and is likely to be rated at 4-7kW). The devices currently used in microwave ovens cost about \$25/kW; hence, no significant cost reductions may be expected from SPS development as the cost target for SPS is \$17-\$20/kW. Efficiency improvements do not promise to be a significant source of benefits either: If a 10 percent increase in energy efficiency were achieved from SPS technology, then the annual benefit from energy saving on the current number of devices would be less than \$2 million per year, assuming a cost of electricity of 60 mills/kWh and a duty cycle of 30 minutes per day.

### 3.3.3 Composite Material Technology

Numerous composite materials such as graphite epoxy composites are starting to find a wide range of commercial applications because of their attractive physical properties, including light weight, resistance to fatigue and crack propagation, vibration damping and strength (especially when high modulus fibers are used). The uses to date are somewhat specialized, involving sports equipment (boats, tennis racquets, golf clubs, face masks) and a limited number of aerospace structural applications (a current high-performance jet fighter uses about 450 kg of composites).

The industry which is most likely to take advantage of composite materials in a significant way in the near term is the surface transportation industry. The motivation for composite use, even though it is more expensive than substitute structural materials, and will be for some time



to come, is the legislative requirements concerning gas mileage in automobiles. It has been estimated\* that consumption of composite materials by the automobile industry to reduce vehicle weight could reach 390 million kg by 1990, from the current consumption of about 110 thousand kg per year. The benefits to be gained from this utilization of graphite composites would include the cost of the gasoline saved from improved mileage and the energy savings in material processing, minus the difference in material costs, if any. If 45 kg of composite materials were used per automobile in 1990 (a conservative estimate), the energy savings in materials processing alone would result in a \$7 million annual benefit. The benefits to be gained from gasoline savings might be more significant, but would require a considerably more sophisticated analysis. Similar, but smaller, benefits might be obtained on other surface transportation industries, such as truck and railroad car manufacturing.

It is important to note that the composite materials used for automotive, aerospace, and sporting applications may not be the materials that would be attractive for an SPS, in that terrestrial applications generally do not require a high-temperature capability which SPS structure might. The majority of currently developed materials are "thermoset" materials like graphite epoxy, which do not have a high-temperature capability. Some thermoset materials with more desirable temperature characteristics have been developed recently, but little information is available about

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\*"Preliminary Economic Evaluation of the Use of Graphite Composite Materials in Surface Transportation," prepared by ECON, Inc., for NASA Headquarters, July 11, 1977.

them. Some of those involved with SPS structural design are more inclined toward "thermoplastic" materials like polysulfone graphite, both because of its temperature properties as well as its on-orbit forming characteristics.\* Although the current cost of polysulfone graphite is about \$180/kg (as compared to \$7.50/kg for sheet aluminum) and is projected to be in the neighborhood of \$40/kg in 1995, it, like many composite materials, requires less energy to form and process on orbit, has superior thermal distortion characteristics, and is electrically insulative. There are a number of other properties beyond cost and mass which need to be considered systematically in the selection of an appropriate structural material for SPS. Such a selection, or at least a narrowing of the field of candidate materials, is necessary before more detailed benefits estimates can be made.

#### 3.3.4 Teleoperators and Other Remote Systems Technology

Remote systems technology similar to that which would be incorporated in the remote-controlled teleoperators used to aid on-orbit construction of SPS units might be useful in aiding manned operations in any hazardous environment or occupation. Several such applications were described in Section 3.1. The primary current application of remote systems technology is in the nuclear power and research industry for the purposes of plant refueling and materials experimentation. If there were an affirmative decision in the United States on nuclear fuel reprocessing, this would serve to maintain the industry at its current level of \$10-\$20 million per

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\*Telecommunication with personnel at Rockwell International, December 2, 1977.

year. This industry is not likely to expand in the near future, as a single nuclear power plant employs only about \$1 million worth of remote systems equipment in a plant which typically costs \$1 billion. The current cutback in nuclear power plant orders will not affect this industry for 6 to 7 years. Currently, there is also a small market for remote devices to be employed in the handling of explosives, rocket fuel, and biologically hazardous materials in research. Also, there is some work, at a very low level, being done in submersible technology; this area could expand significantly when the way is cleared for undersea mineral extraction. This last area of application is the one which most closely parallels developments which would be required for SPS, but it is highly uncertain when or to what extent it might develop. Consequently, the area of remote systems technology may be viewed as currently representing a small potential for benefits from alternative applications.

### 3.4 Recommendations and Conclusions

The solar cell market represents the single most significant area of potential benefit from alternative applications of SPS technologies. It is not clear whether the solar cell material selected for terrestrial application should affect the SPS selection or the reverse, as numerous trade-offs exist in space applications between efficiency, resistance to degradation, specific mass, specific cost, and susceptibility to concentration. A methodology which accounts for economic factors involving terrestrial uses should be developed to aid in the selection of the appropriate solar cell material for SPS.

Trade-offs exist between technical and economic factors in the selection of structural materials for SPS. A methodology, incorporating economic as well as technical factors, to aid in the selection of the proper materials for SPS (including thermal distortion, electrical conductivity, aging characteristics, and on-orbit forming characteristics such as energy requirements), should be developed.

Other technologies which should be examined in more detail for possible benefits from alternative applications include cryogenics and power distribution and switching. A survey of these technologies requires a level of technical detail greater than was possible under this effort. In addition, attention should be directed toward the economic effect of reducing the cost of space transportation from the level attained by the shuttle to that which would be offered by a heavy-lift launch vehicle developed to support SPS.

#### 4. A STUDY OF THE POTENTIAL ELECTRIC POWER MARKET PENETRATION OF SPS

Studies conducted to date have assumed a particular build-up rate of SPS's on-orbit. The assumed rates were chosen from technical considerations without explicit concern for the potential SPS market as a function of SPS cost. The object of this study has been to estimate the potential market for SPS's from strictly economic considerations.

First, it is interesting to examine historical trends in electric energy generation and installed capacity. Figure 4.1 gives installed generating capacity of the U.S. electric utility industry over the past 30 years. Installed capacity increased at an annual rate of 8.4 percent during the period shown and the per capita capacity increased at a rate of 7 percent. During the same period of time, generation capacity, shown in Figure 4.2 increased at an annual rate of 7.4 percent or a per capita annual rate of 5.9 percent. Many projections for future demand and capacity growth focus around these numbers. There is considerable reason to believe, however, that future electric energy and generation capacity growth will be well under the often-quoted 6 to 7 percent per year levels. To begin with, as Figure 4.3 shows, the same period of time has seen a nearly five-fold decrease in the real price of electricity to residential consumers and a two-fold decrease for industrial consumers. If future real prices merely remain constant, one could expect a much-reduced demand growth. In addition, the period of the late 1940s and 1950s was a time of considerable population growth. This trend is now substantially reversed. Figure 4.4 shows the current population forecast through the

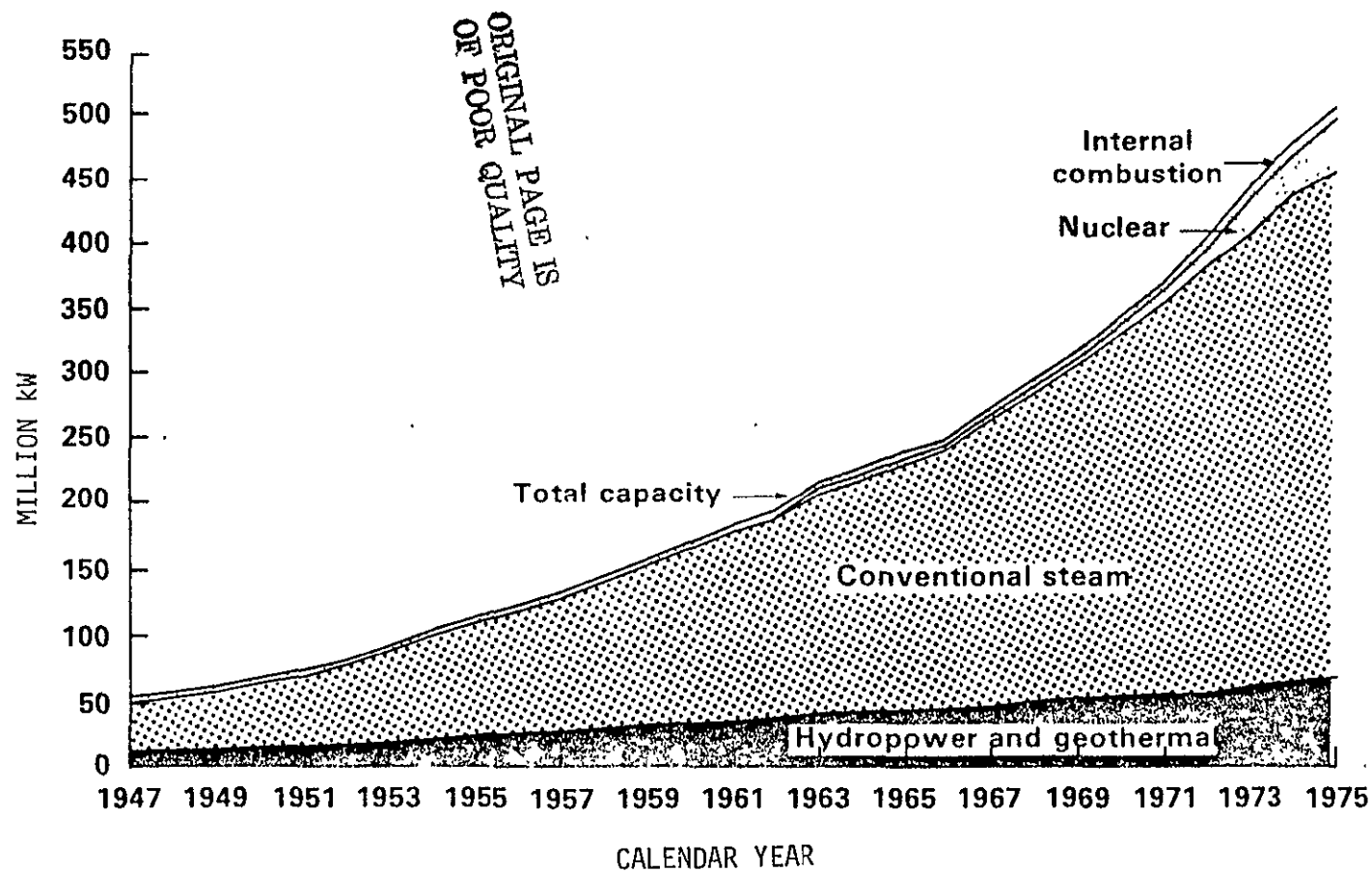


Figure 4.1 Installed Generating Capacity of the U.S. Electric Utility Industry (Source: Energy Perspectives 2, U.S. Department of the Interior, June 1976)

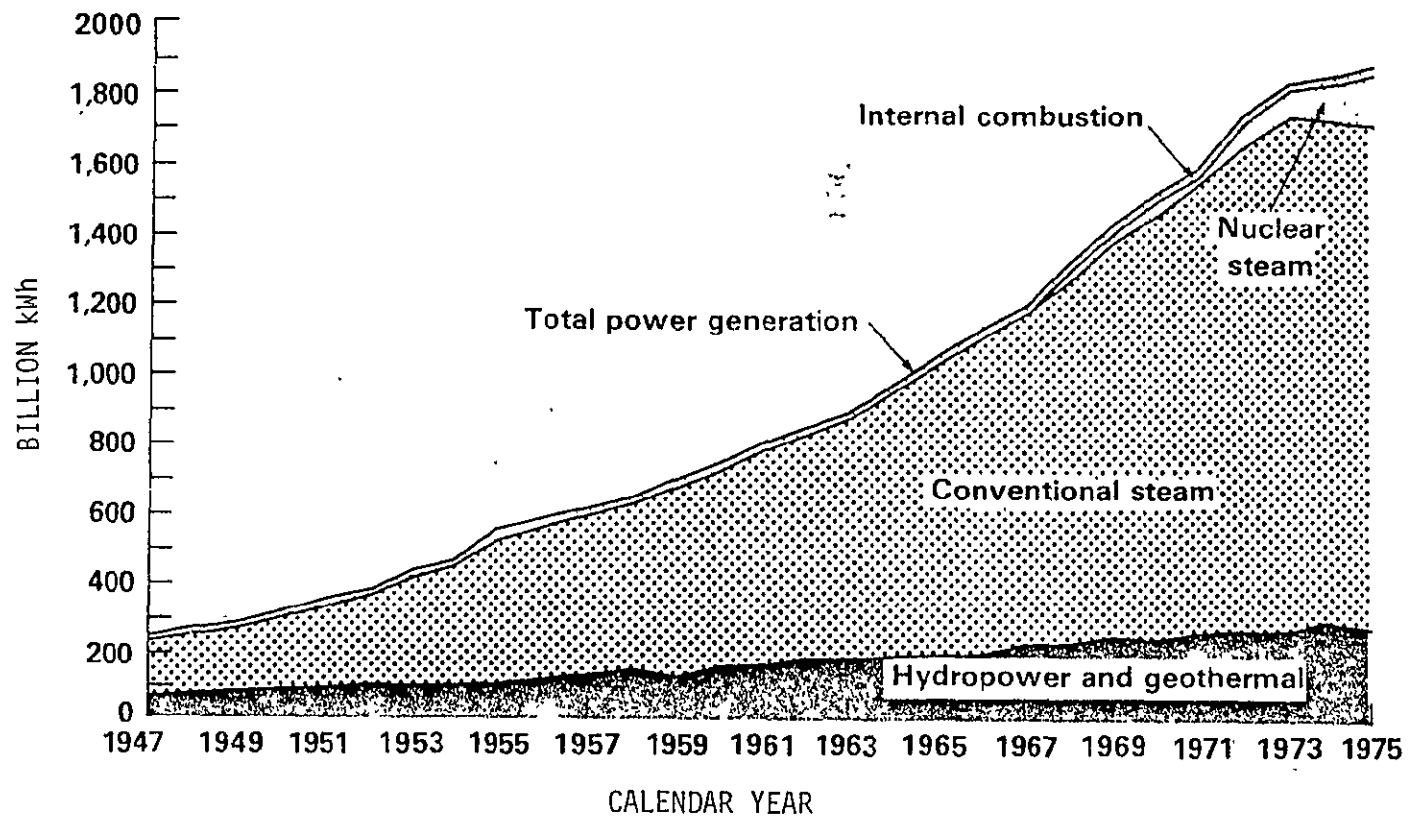


Figure 4.2 U.S. Power Generation by Type of Capacity  
 (Source: Energy Perspectives 2, U.S. Department  
 of the Interior, June 1976)

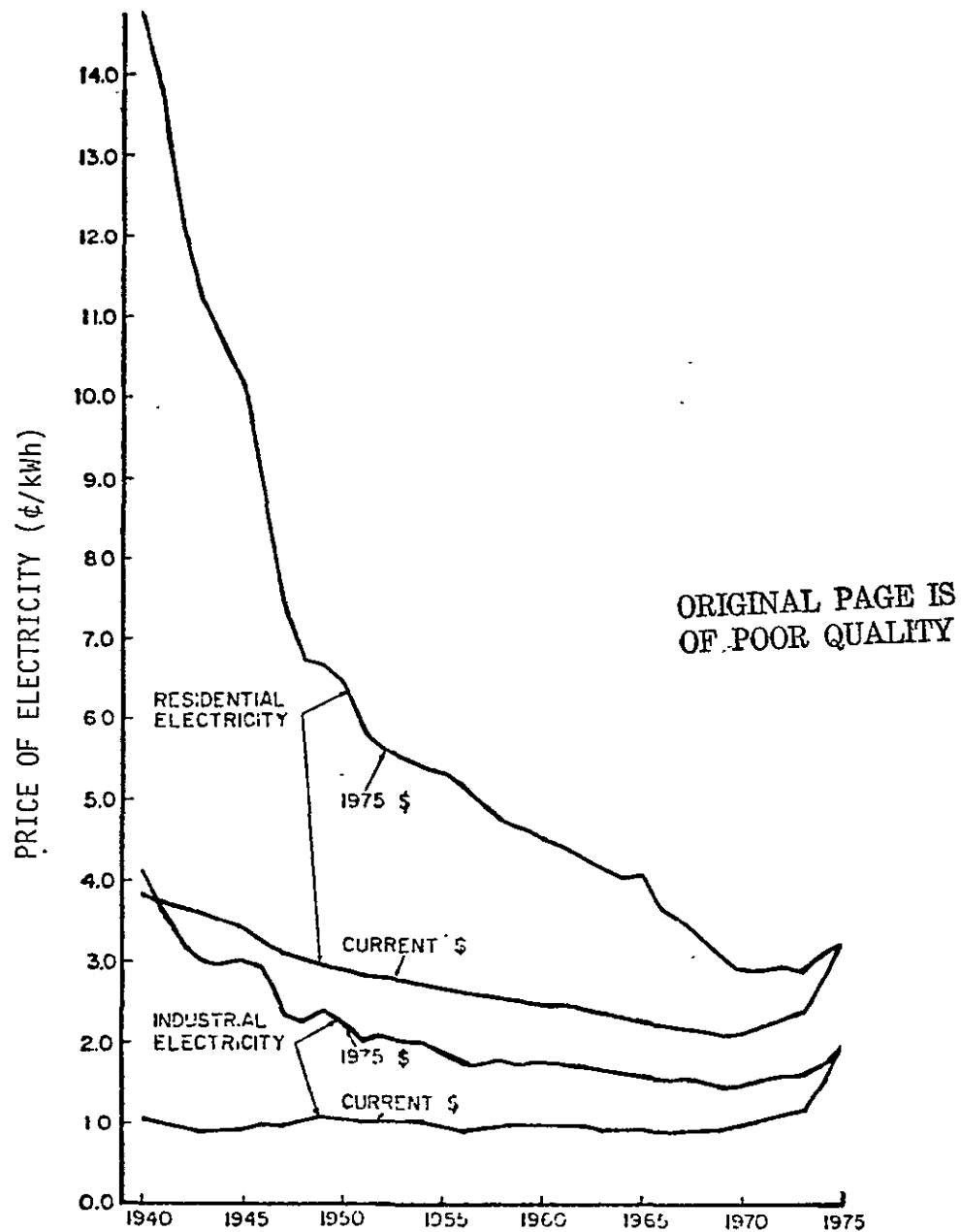


Figure 4.3 The Real Cost of Electrical Power Over the Past 35 Years (Source: Achieving the Goals of the Employment Act of 1946--Thirtieth Anniversary Review, Vol. 2--Energy, Paper No. 2, Energy and Economic Growth, Prepared for the Subcommittee on Energy of the Joint Economic Committee, Congress of the United States, August 31, 1977)



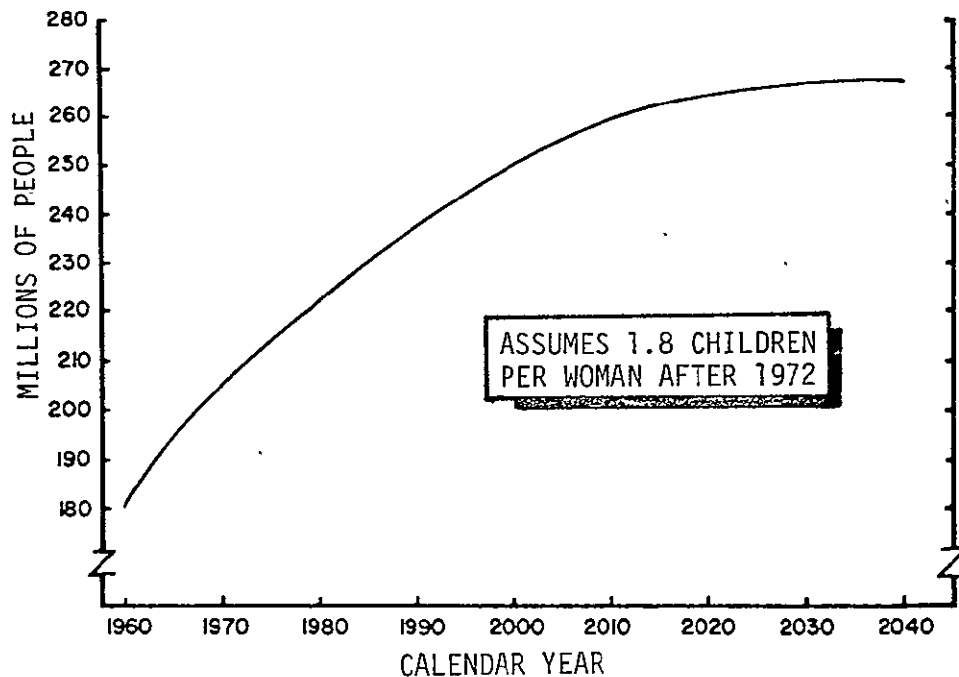


Figure 4.4 Projection of U.S. Population to 2040 (Source: C. E. Whittle, et al., The IEA Energy Simulation Model, A Framework for Long-Range U.S. Energy Analysis, Institute for Energy Analysis, ORAU-125 (IEA75-1), January 1976)

year 2040. The present rate of population growth in the United States is only 0.7 percent per year.

If the real price of electricity remains constant, or perhaps even increases slightly, there is little reason to believe that per capita consumption of electric energy should increase substantially unless the nation undertakes a major shift from fossil fuels to an all-electric economy. This is likely only if three conditions are met:

1. New, low cost, non-fossil fuel energy sources are found for electric power generation
2. Efficient, light-weight energy storage devices are developed
3. The real price of fossil fuels increases substantially.

Then allowing for per capita growth in energy demand as shown in Figure 4.5, a 7 percent per year growth in electrical energy demand corresponds to a transition to an all-electric economy over about the next 50 years. This is, of course, quite unlikely, especially insofar as this would leave a considerable supply of easily recoverable fossil fuels which would then be priced very competitively.

A more likely scenario entails a rather modest shift toward an all-electric economy, but with, perhaps, even a decrease in per capita total energy consumption. This could lead to an electric energy demand growth as low as 2 percent per year (perhaps even lower), but most likely not more than 4 percent per year. A 4 percent per year demand growth is the level presently assumed by the Philadelphia Electric Company for in-house planning purposes.

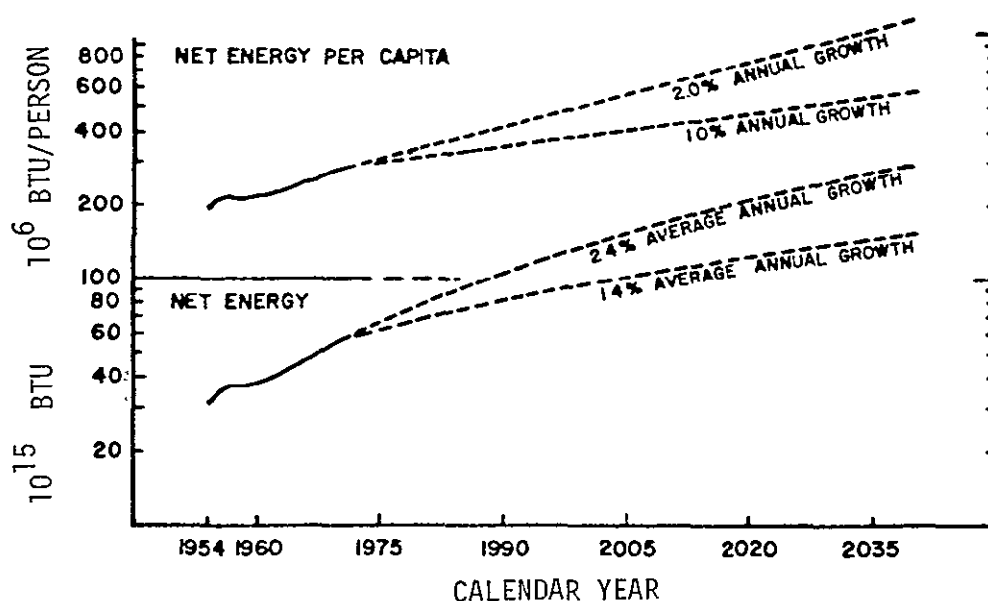


Figure 4.5 Trends and Projections for Net Energy and Net Energy Per Capita

The potential market for SPS derives from two demand components: the market generated by new baseload energy demand and the market generated by retiring capacity. Figure 4.6 shows the potential SPS market generated by new baseload energy demand. It is interesting to observe that, at the lower demand growth rates, the market would not support the construction of two 5 GW SPS units per year until somewhat after the year 2000. When one further considers that this growth would be spread over the entire nation, it becomes apparent that a 5 GW system could very possibly encounter some difficulty in penetrating this market.

The capacity demand created by retiring generation capacity is shown in Figure 4.7. This demand could be considerable. However, it is somewhat dangerous to assume that this entire demand would be subject to capture by SPS. First, recognize that this demand is created by retiring existing plants. Thus, there may be a considerable impetus to refurbish or rebuild these plants rather than to decommission them. Second, due to the recent, rapid growth in generation capacity, the majority of existing capacity will retire in the 2000 to 2010 period. With up to 35 GW capacity being retired each year, it might not be either possible or desirable to replace this entire capacity with SPS. The odd shape of the retiring capacity curve of Figure 4.7 is, in fact, due to the recent high growth rate followed by a period of lower growth. The dashed line on this figure shows the potential retirement of capacity installed after 1975, which continues to increase rapidly. This raises the issue of new capacity lifetime. If new capacity has a lifetime of, say, 40 years, there would appear a period, in this case of 10 years duration, during which the retiring capacity would be relatively low.

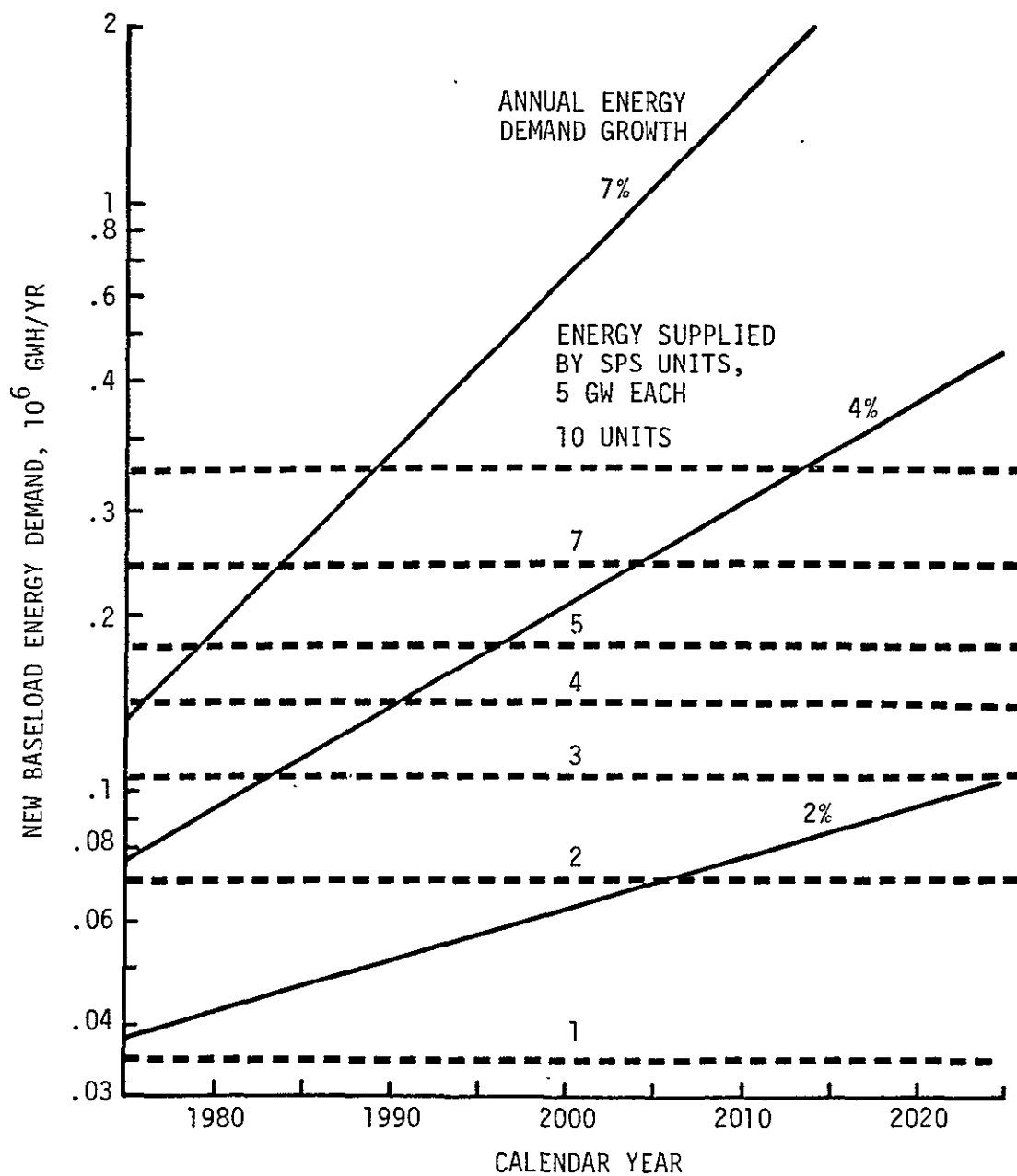


Figure 4.6 Potential SPS Market Generated by New Baseload Energy Demand

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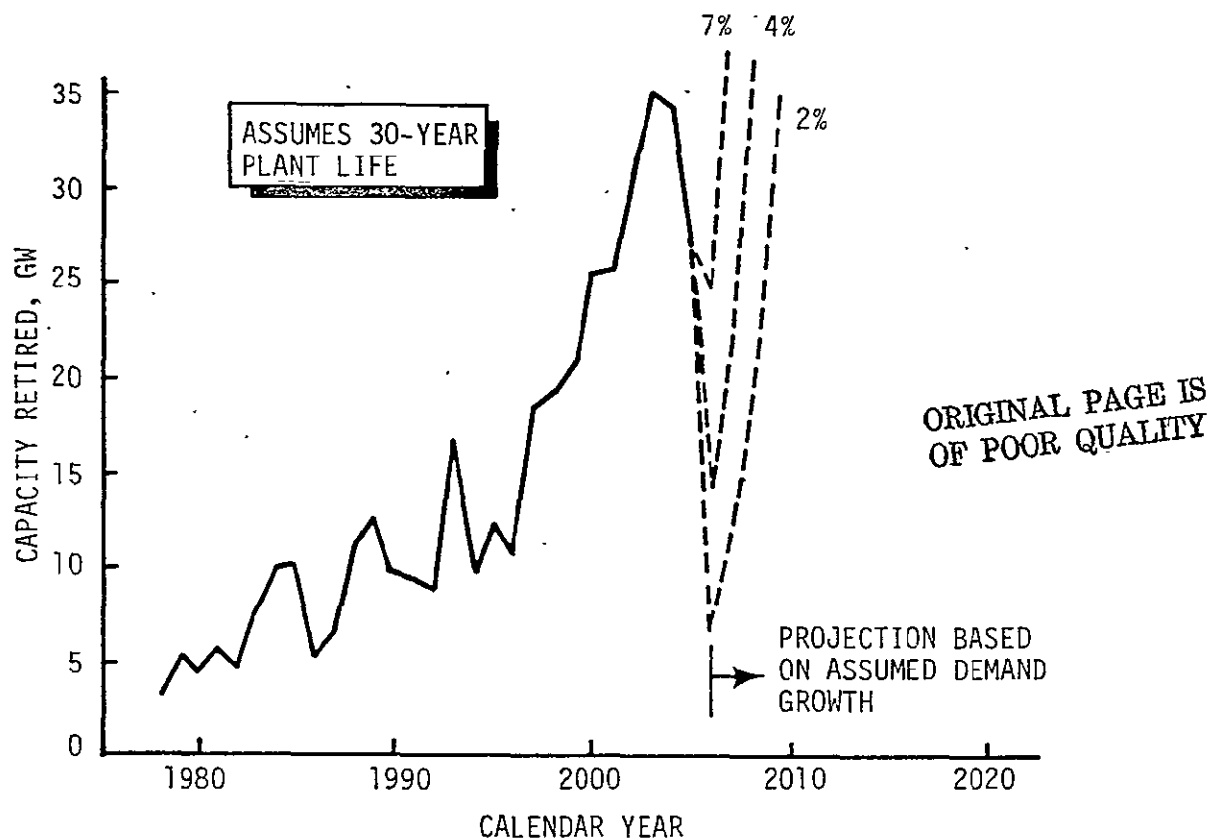


Figure 4.7 Annual Capacity Retired

In general, a power plant does not run at a high capacity factor (say 63 percent) up to retirement but, after reaching a peak typically about 10 years into its life, gradually produces less and less energy until, in its thirtieth year, its capacity factor is about 10 percent. Thus, capacity retires gradually over as much as 10 to 20 years. Part of the reason for this lies in the economics of operating old versus new equipment. This effect must be taken into account when estimating the potential SPS market.

With this background it is now possible to model the electric utility industry to determine the potential SPS market. The computations

presented have been made using the ALPS (A Linear Programming System for Forecasting Optimum Power Growth Patterns) computer program developed by the Hanford Engineering Development Laboratory.\*

The ALPS program has been written to compute the optimum growth pattern of fossil and nuclear electric power plants. Plants are selected to minimize the total discounted costs while satisfying constraints on energy demand, fuel availability, introduction dates, and construction rates. The ALPS model consists of the MAJOR matrix generator code, any generalized LP code, and the SCRIBE report writer code.

The matrix generator code, MAJOR, reads input from cards and/or tape, computes and prints the matrix coefficients, and writes the coefficients on tape in a format compatible with the LP code. The LP code reads the coefficients from tape, computes the optimum solution, and writes the solution on another tape. Finally, the report writer code, SCRIBE, prints the optimal solution and miscellaneous information using data from cards and the MAJOR and LP output tapes. Although the input/output of all three codes are mutually compatible, each code is distinct, and must be executed separately. This permits the ALPS system to employ any generalized LP code.

The nature of the problem solved can be characterized as follows. For each stage, an energy demand or, equivalently, a new capacity demand, is specified. ALPS then builds, at each stage, the plants that:

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\* R. W. Hardie, W. E. Black and W. W. Little, ALPS, A Linear Programming System for Forecasting Optimum Power Growth Patterns, Hanford Engineering Development Laboratory, HEDL-TME 72-31, April 1972.

1. Meet the energy demand
2. Satisfy constraints on fuel availability, plant availability, etc.
3. Minimize the total discounted costs.

Constraints include Pu and U-233 availability and, by plant, scheduled capacity additions, plant availability (i.e., advanced plants are not available until a specified date), and maximum buildup rates.

The total system cost is composed of the total plant costs (excluding fuel), fuel costs, and taxes. Fissile Pu and U-233 costs are not included in the fuel cost since they are generated and used internal to the system.  $U_3O_8$  costs, however, are included, and are computed as a function of the quantity of  $U_3O_8$  consumed. Fossil fuel costs are also included, and simply read-in as input data. All costs are discounted from their occurrence to the beginning of the initial stage.

The simulation results are shown in Figure 4.8, expressed in terms of the potential demand for new SPS units. The potential demand assumes that all cost-effective opportunities for SPS implementation are captured. This includes both new demand and retiring capacity demand. Typically, the "next best" system is the Canadian heavy water reactor, CANDU. CANDU capital costs are about 10 percent higher than typical light water reactor costs and their fuel costs are quite low since they do not require enriched uranium. The crossover cost between CANDU and SPS would appear to be on the order of \$1500/kW (1975 dollars) installed capital cost for SPS (plus or minus about 30 percent, depending on future regulatory constraints imposed on CANDU). The break-even cost becomes somewhat higher also if SPS lifetime is increased beyond 30 years and/or if discount rates lower than 7.5 percent are used for the comparison.

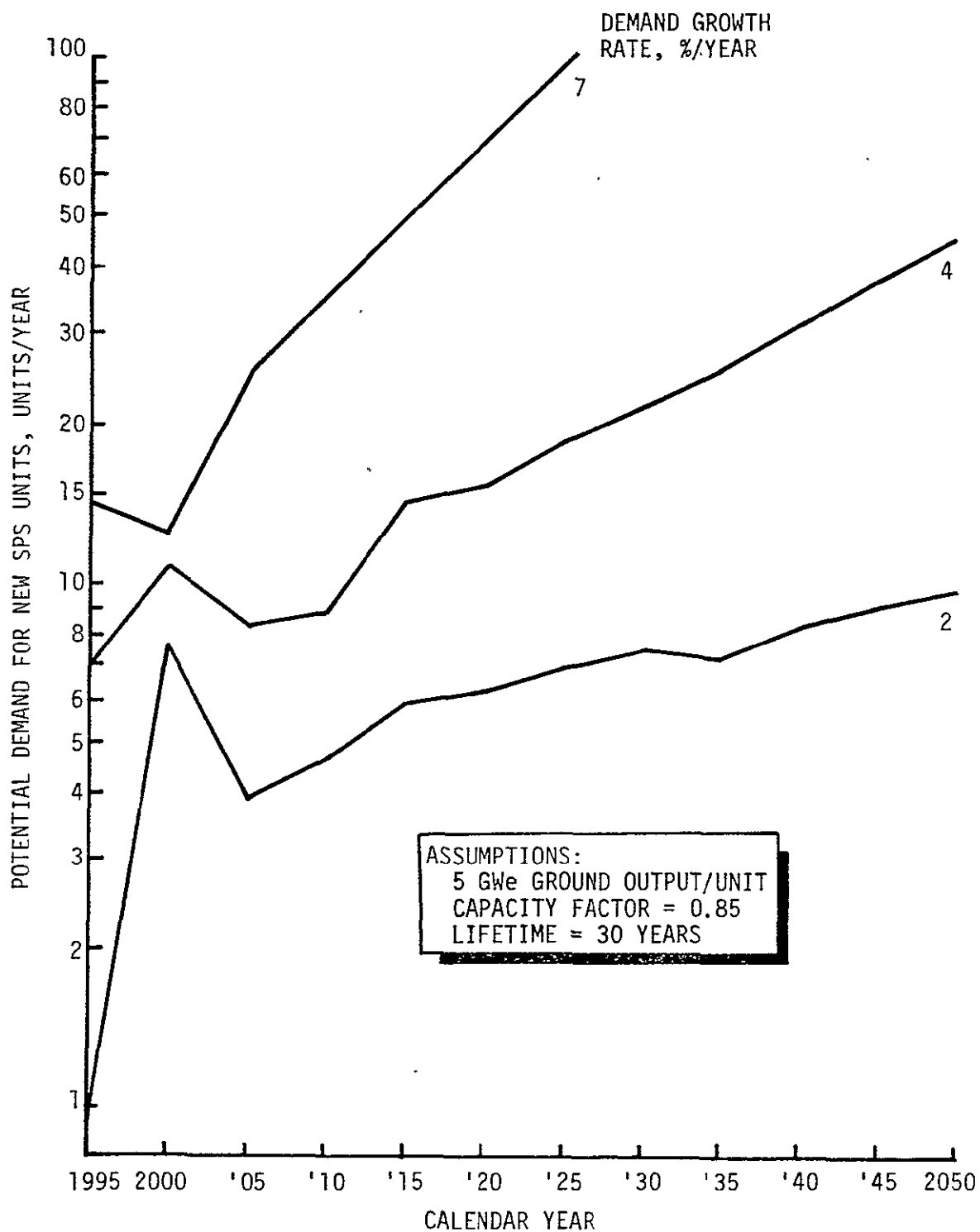


Figure 4.8 The Potential Demand for SPS Units



The conclusions of this study are that a U.S. market of one-to-two new 5 GW SPS units per year after the year 2000 could be generated by electric power demand growth rates as low as 2 percent per year. A somewhat higher demand growth rate would likely be necessary to provide a market for three or more 5 GW SPS units per year. Such higher demand growth rates would probably correspond to a rather rapid swing to an all-electric economy. Cost-effectiveness of SPS will be determined by comparison against nuclear plants. The key competing nuclear plants will likely be CANDU and breeder reactor. However, if political constraints are imposed upon the implementation of nuclear plants, the cost-effective cost of SPS would probably be determined by environmentally acceptable coal plants. Besides the capital and operating costs of competing systems, the key variables which determine the cost-effective cost for SPS are SPS lifetime, discount rate, and capacity factor. These variables need further study.

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# APPENDIX A

## DERIVATION OF OPEC FUEL PRICING MODEL

This appendix develops the solution to (2-1) in Section 2.2. Consider the problem of selecting a functional  $q(t)$  so as to maximize  $J$ ,

$$J = \int_0^T U[q(t), Q(t), t] e^{-\rho t} dt,$$

subject to the constraints:\*

$$\dot{Q}(t) = -q(t)$$

$$q(t), Q(t) \geq 0,$$

We will specify a priori the following conditions on  $U$ :

$$U_q, U_Q > 0$$

$$U_{qq} \leq 0,$$

and that  $Q(t_0) = Q_0$ . These conditions, in particular that on  $U_q$ , guarantee that

- (i) the control inequality constraint  $q \geq 0$  will never be active, and
- (ii) there exists some time,  $T$ , where  $Q = 0$  and the state inequality constraint  $Q \geq 0$  does become active.

Although the problem is properly considered as an infinite horizon one, the nonrenewability of the resource  $Q$  and condition (ii) above make it such that the control problem ends at time  $T$ ; there exist no more options.

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\*The following notations are used:  $\dot{x} = \frac{dx}{dt}$  and for  $F(x, y)$ ,  $F_x = \frac{\partial F}{\partial x}$ .  
By  $\hat{x}(t)$  is meant the optimal time path of  $x$ .

From Takayama<sup>\*</sup> (Theorem 8.A.3, p. 613), the necessary conditions for the optimality of the functional  $\hat{q}(t)$  are:

$$(i) \quad \dot{Q} = \frac{\partial \hat{H}}{\partial s}, \quad \dot{s} = - \frac{\partial \hat{H}}{\partial Q},$$

$$\text{where } \hat{H} = H[\hat{q}(t), \hat{Q}(t), t, s(t), v(t)]$$

$$= v(t)u[\hat{q}(t), \hat{Q}(t), t]e^{-rt} - s(t)\hat{q}(t);$$

$$(ii) \quad H[\hat{q}(t), \hat{Q}(t), t, s(t), v(t)] \geq H[q(t), \hat{Q}(t), t, s(t), v(t)], \text{ for all admissible } q(t);$$

$$(iii) \quad H[\hat{q}(T), \hat{Q}(T), T, s(T), v(T)] = 0;$$

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$$(iv) \quad v(t) = \text{constant} \geq 0.$$

$s(t)$  and  $v(t)$  are Lagrangian multipliers or adjoint variables.

Condition (ii) requires

$$vU[\hat{q}(t), \hat{Q}(t), t]e^{-\rho t} - s(t)\hat{q}(t) \geq vU[q(t), \hat{Q}(t), t]e^{-\rho t} - s(t)q(t),$$

given the constant  $v$  as specified in condition (iv). Now if  $v = 0$ , then

$$s(t)\hat{q}(t) \leq s(t)q(t).$$

Since  $v$  and  $s(t)$  cannot vanish simultaneously ("Fritz John's theorem"; see Takayama, p. 612),  $s(t) \neq 0$ . If  $s(t) < 0$ , this implies  $\hat{q}(t) \geq q(t)$  for all  $q(t) \geq 0$ , and thus  $q(t)$  would be unbounded above. But an infinite rate of drawdown is clearly not optimal.<sup>\*\*</sup> If  $s(t) > 0$ , and

<sup>\*</sup>Takayama, A., Mathematical Economics, The Dryden Press, 1974.

<sup>\*\*</sup>That an infinite rate of drawdown were optimal would imply that buyers could absorb  $Q_0$  in an infinitesimal amount of time; and this would occur without letting demand become more elastic than -1.

letting  $q(t) = 0$ , the implication is that  $\hat{q}(t) = 0$  for all  $t$ , which is clearly not optimal given the condition  $U_q > 0$ . Thus,  $v \neq 0$ . Without loss of generality we can set  $v = 1$ . Condition (ii) also yields the condition  $\frac{\partial H}{\partial q} = 0$  up to time  $T$  since  $q(t)$  will be in the interior of the admissible region.

From the above conditions, the following equations are obtained:

$$\dot{s} = -U_q e^{-\rho t} \quad (A-1)$$

$$\dot{Q} = -q(t) \quad (A-2)$$

$$s(t) = U_q e^{-\rho t} \quad (A-3)$$

$$s(T)\hat{q}(T) = U[\hat{q}(T), \hat{Q}(T), T]e^{-\rho T} . \quad (A-4)$$

Equations A-1 to A-3 describe the movement of the system up until the entry into the constraint, i.e.,  $0 \leq t \leq T^-$ . Equation (A-4) describes the transversality condition at  $T^+$ . (A-1) to (A-4) is a system of two first-order differential equations, (A-1) and (A-2), with two boundary conditions known:  $Q(t_0) = Q_0$  and  $Q(T) = 0$ . The system also contains the unknown  $T$  and two independent equations, (A-3) and (A-4). Henceforth, it will be understood that  $q(t)$  and  $Q(t)$  refer to the optimal time paths.

We have not as yet considered the effect of the state variable inequality constraint,  $Q(t) \geq 0$ . Doing so will yield an additional necessary condition for optimality, called the jump condition, and we

seek an expression for  $\dot{s}(T)$ . According to Pontryagin et al.,\* at the time that the optimal path enters into the constraint boundary [here, time  $T$  where  $Q(t) = 0$ ], the following condition holds:

$$\left. \begin{aligned} v^+(T) &= v^-(T) \\ s^+(T) &= s^-(T) \end{aligned} \right\} \quad (A-5)$$

(A-5) permits the equating of  $s^+(T)$  from (4) with  $s^-(T)$  from (A-3).

Before we proceed further, we must specify the form of  $U[q(t), Q(t), t]$ .

#### The Increasing Demand Case

Let us consider a demand function in which the quantity demanded as a function of price grows exponentially. Such a function has the following form:

$$q(t) = \left[ \frac{p(t) - a_1}{a_2} \right] e^{\gamma t}.$$

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Solving for  $p(t)$  yields

$$p(t) = a_1 + a_2 e^{-\gamma t} q(t) \quad (A-6)$$

Costs are given as

$$c(t) = b_1 q(t) + \frac{b_2}{2} q(t)^2 + b_2 [Q_0 - Q(t)] q(t). \quad (A-7)$$

Let us now define the (undiscounted) rate of utility increase as net revenue:

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\* L. S. Pontryagin, V. G. Boltyanskii, R. V. Gramkrelidze, and E. F. Mishchenko, The Mathematical Theory of Optimal Processes, New York, Wiley Interscience, 1962 (tr. by K. N. Trirogoff from Russian original), p. 302.

$$\begin{aligned}
 U[q(t), Q(t)] &= q(t) \cdot p(t) - c(t) \\
 &= (a_1 - b_1 - b_2 Q_0)q(t) + \left( a_2 e^{-\gamma t} - \frac{b_2}{2} \right) q(t)^2 + b_2 Q(t)q(t),
 \end{aligned}$$

or, simplifying coefficients,\*

$$U[q(t), Q(t)] = C_1 q(t) + h(t)q(t)^2 + C_3 Q(t)q(t). \quad (A-8)$$

We now obtain the following

$$U_q = C_1 + 2h(t)q(t) + C_3 Q(t) \quad (A-9)$$

$$U_Q = C_3 q(t). \quad (A-10)$$

Substitute (A-9) into (A-3)

$$s(t) = [C_1 + 2h(t)q(t) + C_3 Q(t)]e^{-\rho t} \quad (A-11)$$

and solving for  $q(t)$

$$q(t) = \frac{1}{2h(t)} [s(t)e^{\rho t} - C_1 - C_3 Q(t)] \quad (A-12)$$

Consider now equation (A-4) which describes the transversality conditions.

Substituting (A-8)

$$s^+(T)q(T) = -q(T)[C_1 + h(T)q(T) + C_3 Q(T)]e^{-\rho T}.$$

Now, if  $q(T) \neq 0$ , divide through by  $q(T)$ , noting that, by definition  $Q(T) = 0$ :

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\* Although we have specified that  $h(t) = a_2 e^{-\gamma t} - \frac{b_2}{2}$ , there is no actual restriction on the form of  $h(t)$  in what follows.

$$s^+(T) = -C_1 e^{-\rho T} - h(t) e^{-\rho T} q(T).$$

Substitute (A-11)

$$s^+(T) = -C_1 e^{-\rho T} + \frac{1}{2} s^-(T) + s^-(T) + \frac{1}{2} C_1 e^{-\rho T}$$

Noting that  $s^+(T) = s^-(T)$  (the jump condition),

$$s^-(T) = C_1 e^{-\rho T}.$$

If, on the other hand,  $q(T) = 0$ , we solve (A-11) directly for  $s^-(T)$ , finding

$$s^-(T) = C_1 e^{-\rho T}. \quad (A-13)$$

Thus we have obtained our final necessary condition for optimality of the Constant Demand Case.

We can now proceed to solve the system of differential equations.

Recalling  $\dot{Q} = -q$ , differentiate (A-11) to obtain

$$\begin{aligned} \dot{s} = & -\rho e^{-\rho t} [C_1 - 2h(t)\dot{Q} + C_3 Q(t)] \\ & + e^{-\rho t} [-2h'(t)\dot{Q} - 2h(t)\ddot{Q} + C_3 \dot{Q}] \end{aligned} \quad (A-14)$$

Substitute (A-10) into (A-1)

$$\begin{aligned} \dot{s} = & -C_3 q(t) e^{-\rho t} \\ = & C_3 \dot{Q} e^{-\rho t} \end{aligned} \quad (A-15)$$

Equate (A-14) and (A-15)

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$$-\rho C_1 + 2\rho h(t)\dot{Q} - \rho C_3 Q(t) - 2h'(t)\dot{Q} - 2h(t)\ddot{Q} + C_3\dot{Q} = C_3\dot{Q}$$

Thus,

$$-2h(t)\ddot{Q} + 2[\rho h(t) - h'(t)]\dot{Q} - \rho C_3 Q(t) = \rho C_1 \quad (A-16)$$

or

$$\ddot{Q} - \left[ \rho - \frac{h'(t)}{h(t)} \right] \dot{Q} + \frac{\rho C_3}{2h(t)} = - \frac{\rho C_1}{2h(t)} \quad (A-17)$$

Thus, we have obtained a second-order ordinary differential equation in  $Q(t)$ . We know the boundary conditions  $Q(0) = Q_0$  and  $Q(T) = 0$ . Additionally we know from (A-13) and (A-11) that  $q(T) = 0$ . We do not, however, know  $T$ . This equation is solved numerically by beginning at some estimated  $T$ , letting  $Q(T) = q(T) = 0$  and solving (A-17) backwards until  $t = 0$ . At that point the numerically estimated  $Q(0)$  is checked against  $Q_0$  and, if different, a new  $T$  is tried. This process is performed iteratively until a correct  $T$  is found and the optimal  $Q(t)$  functional is calculated.



## APPENDIX B

ABC's ISSUES AND ANSWERS INTERVIEW WITH  
THE SHAH OF IRAN--November 20, 1977

- Q: Lets turn to the question of oil. It has been reported that you have told the President that you will not ask for or be a leader in asking for price increase at the meeting of the OPEC nations next month in Venezuela. Is that correct?
- A: That is correct.
- Q: In the past you have asked or have been one of the leaders for increase of prices, why have you changed your position?
- A: Well you are saying, Ms. Walters, that we were a leader for asking for increases, but we think that even when we were asking for increases we were adopting the moderate line; there were much harder people then.
- Q: Alright, let us give you that. But this is still a somewhat different position.
- A: OK. Now, we are told that the world economy is in poor shape, that we shall have no advantage in damaging it still further. Agreed. But for how long can you postpone the real problem which is to find alternative resources for energy; I mean oil. So I am ready, as far as my country is concerned, to take that chance and that responsibility to go along with you, to give you a spell, to give you a break, provided you also take the necessary measures and steps for conservation and finding your sources of energy.
- Q: How long a break?
- A: As short as possible, even if I consider your own interests.
- Q: If the other countries should push for a price increase in Venezuela and so forth, would you acquiesce or would you fight it?
- A: Well, I don't think they could do it without our participation or the Saudis.
- Q: The question here is really whether you would go along with the small increase. President Carter has asked for a freeze on oil prices, no increase at all.
- A: I wouldn't mind that.

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Q: You would support. Would you oppose the Saudis or the other moderates who want an increase of somewhere between 5 and 8 percent? Would you oppose that and go along with President Carter's request for an actual freeze on the present price?

A: I don't think the Saudis would push for an increase.

Q: Not even for a 5 percent?

A: I don't think so.

Q: And how about some of the other moderate countries such as Venezuela has been mentioned, Kuwait, Indonesia who would want a small increase. Would you oppose them?

A: They could not carry the ball by themselves.

Q: So you think it will be price freeze?

A: Could be.

***ECON Corporate Headquarters:***

**Princeton, New Jersey  
Telephone 609-924-8778**

***Western Office:***

**San Jose, California  
Telephone 408-249-6364**